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Chapter 4 A pattern based approach to design research in technology-enhanced mathematics education

This chapter responds to Aim 1 identified in Chapter three: To identify potential elements of an epistemic infrastructure for a design science of technology enhanced mathematics education. The proposed elements include a cycle of enquiry, design narratives, and design patterns. These constructs are reviewed and critiqued. The next chapter uses them as a basis for a methodological framework.

4.1 Introduction: In Search of an Epistemic Infrastructure for a Design Science of Mathematical Education

Chapter two put forth a case for a design science of mathematics education, reviewed the existing tradition in this paradigm, and highlighted some theoretical and methodological challenges. An observation of the state of the field suggested a need for a clearly articulated consensual epistemic infrastructure. Some requirements for this infrastructure where identified, in terms of accessibility, transparency and traceability, expressiveness, orientation, and cumulativity. These requirements gave rise to several open questions (section 2.4.4):

- How do current methodological tools map to these requirements? Can they be combined to provide a more comprehensive coverage?
- Which new tools can be incorporated to address the issues noted and how?
- Once a comprehensive epistemic infrastructure is identified, properly articulated and justified, will it fulfil the promise of a design science of education?

This chapter offers a contribution towards the overarching goal of establishing an epistemic infrastructure, by addressing some of these questions. I begin by describing two consolidated cycles of design research, drawing on several expert accounts (Middleton et al, 2008; Gravemeijer and Cobb, 2006; Lesh and Sriraman, 2005; Cobb et al, 2003) reviewed in Chapter 2. In line with these accounts and others, this cycle is situated within a broader framework.

Each stage in these cycles calls for appropriate methodological tools. I mention a few and discuss their relative merits and challenges. I then focus on a specific set, including design narratives and design patterns. I conclude with some challenges associated with these.

4.1.1 Common Cycles of Scientific Process

As discussed in Chapter Two, design research in mathematics education is commonly described as a cyclic process. At the core of this process is the design experiment, which oscillates between theoretical and practical innovations. Most authors situate the cycle of design experiment within the context of their research setting. However, when the various descriptions are compared and the contextual details blurred, a stable image emerges (Figure 1).

As in most scientific endeavours, a design experiment would typically start from a theoretical stance, which the researcher would project into a particular problem domain to derive a conjecture. This conjecture is examined by designing artefacts (tools and practices) which embody it. The artefacts are implemented and used in action, ideally in a realistic educational setting. The researcher collects evidence of the successes and tensions arising from the use of the artefacts, with respect to the learning aims. This evidence is interpreted, analysed and evaluated, and the results fed back into an updated theoretical stance.

Although scientific enquiry often stems from a theoretical stance, the cyclic nature of design experiments suggests that other options are just as valid, e.g. a study led by a teacher and originating from her classroom experience.

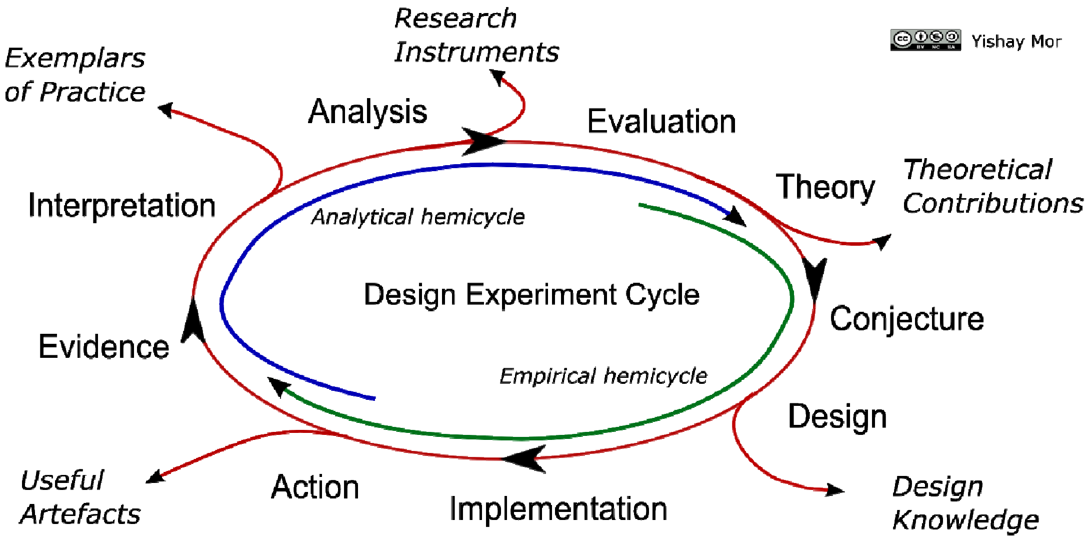

Figure 1: the design experiment cycle, synthesized from the accounts of several experts in the field. Research conjectures are derived from theory, and explored by design, implementation and field testing of new artefacts (tools and activities). The evidence from the field trials is collected, interpreted, analysed and evaluated – feeding back into theory and on to the next iteration.

As argued in Chapter two, a design science of education peruses a double-edged agenda to produce theoretical as well as practical innovations. Figure 1 illustrates how these aims are reflected in the outputs derived from the different phases in the cycle. The empirical hemicycle proceeds from theory to action through design and implementation, ultimately producing artefacts (technological tools, curricular materials, teaching methods, etc.) which should be useful for practitioners operating in similar situations. The evidence collected from the action phase and its interpretation produce exemplars of practice, which provide practitioners with valuable insights as to how to make effective use of the artefacts. As noted by Schwartz et al. (2008), the study of innovative artefacts demands innovative research instruments. The development of such instruments is an inevitable by-product of the interpretation and evaluation phases. Some of these instruments are specific to the situation being studied, but others are useful for peers studying similar situations.

The pinnacle of the analytical hemicycle, starting from the end of the action phase, is in a contribution to an updated theoretical stance. This contribution has two facets: a reflection on the underlying theory, validating or challenging the premises of the experiment, and local theories and ontological (diSessa and Cobb, 2004) or epistemic innovations (Schwartz et al., 2008) referring to the specific problem domain. These theoretical innovations feed back into the design process, along with direct input from the analytical outcomes. The outputs of the design phase are representations of design knowledge which reflect a projection of theory into the problem domain, and adjusted to meet the pragmatic constraints imposed by the learning context. Such design knowledge, if properly represented, should be valuable beyond the unique situation being studied.

Design experiments are embedded in a broader cycle of design research (Middleton et al., 2008; Gravemeijer and Cobb, 2006; Pratt, 1988). This cycle include a preliminary phase where the research problem is framed, an empirical phase consisting of an iterative design experiment, like that described above, and a longitudinal reflective phase of retrospective analysis (Figure 2).

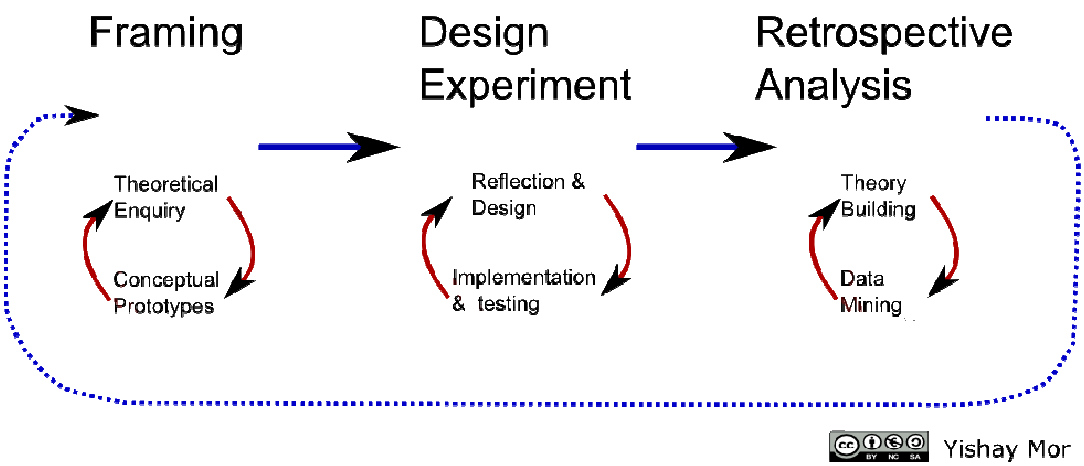

Figure 2: design research meta-cycle, synthesized from the accounts of several experts in the field. The life cycle of a design study begins with a framing phase, iterating between theoretical enquiry and prototyping. This is followed by multiple iterations of the design experiment cycle described in Figure 1. The study is concluded by a retrospective analysis phase, considering data from across the multiple empirical iterations.

The framing and retrospective analysis phases are by and large context independent; they are conducted by the researcher at the comfort of her desk and thus are not constrained by the experimental setting. In fact, the retrospective analysis does not differ significantly in structure from similar phases in other research paradigms, although the actual sources of data and methods of analysis do. The framing phrase does reflect the unique nature of design science, in its underlying premise of the link between knowledge representation and artefacts. This principle, which underlies many of the research questions, also motivates the researchers' own process of understanding and interpreting theory. The framing phase oscillates between reviewing existing literature for theoretical concepts and reifying these concepts in quick prototypes. Such prototypes are primarily used as a means of understanding the theory, as a form of "armchair experiments", and would most often be discarded before the next phase.

The design experiment phase iterates along the path described above. The number and nature of iterations varies, but they are expected to be expansive; each iteration extends the scope or validity of the previous ones. At some point the cycles of the design experiment are concluded, and the study shifts to a retrospective analysis, taking in a long perspective covering multiple iterations and calibrating with other studies. This phase is also iterative; building theories and mining the history of the project for supporting data. In reality, the boundaries between the three phases are often blurred. Pratt (1998) describes a four-iteration structure used in his study:

- **Iteration 0: Bootstrapping**. A 'blue sky' phase, free-form exploration of the platform and the tools it offers. Collecting initial reactions from a pair of children provides preliminary ideas as to what may work.
- **Iteration 1: Exploratory**. Initial development and testing of tools within the platform / environment. The emphasis is still on basic usability and intuitive indicators of learning potentials.
- **Iteration 2: Developmental**. The designs have achieved a level of maturity which allows the researcher to shift attention to questions of learning and specific aims within a well-defined educational context and content domain.
- **Iteration 3: Analytical**. Relatively minor changes to design, and careful attention to questions of data collection, analysis and theorisation.

Each iteration in Pratt's framework consisted of two steps: design-development and use-evaluation. The lessons learnt in the second step of each iteration defined the agenda for the first step of the next. Such a structure is typical of many design based studies in education. It reflects the emergence in tandem of the researcher's own understanding of the tools he or she develops and the evidence collected regarding the influence these tools have on students' understanding of the subject matter. Pratt's iteration 0 may overlap with the framing phase, and iteration 3 leads naturally into the retrospective analysis.

The two cycles presented here provide a "wireframe" for an epistemic infrastructure for design research in mathematics education. In order to complete the model, we need to identify suitable forms of representing knowledge in each node of these cycles, and the means for moving from one node to the next. These would include research methods and "translation schemes" for converting knowledge from one representation to another. Together, these would provide the fabric of argumentative grammar.

The next section considers a possible path from experience to formal knowledge. This model derives elements of a scientific methodology from a naturalistic process of knowledge construction.

4.2 A Possible Path from Experience to Knowledge

The empirical hemicycle of a design experiment has a variety of analogues to refer to. The act of deriving conjectures by projecting theory into context is common to most scientific practices, whereas the act of expressing design knowledge in artefacts is rooted in the crafts of teaching and educational development. Although there is a lot to be explored in terms of formalising these arts, it is the analytical hemicycle that warrants primary attention. This hemicycle is unique to design research, and consequently less documented and farther from intuition.

This section seeks to articulate a framework for the analytical hemicycle, by reference to an innate process by which we extract knowledge from experience. The main motivation for grounding a scientific framework in innate epistemic dynamics is lucidity, the main challenge is rigour; tracing a natural process should make it easier to understand but since such a process does not emerge from scientific tradition, it runs the risk of compromising standards of research This process, illustrated in Figure 3, starts from our physical presence in the world. As we experience events, we compare them to our previous memories. This comparison takes place at a perceptual level. Recurring perceptions arouse episodic memories of past events, while novel experiences stimulate our attention (Biederman and Vessel, 2006). In our attempt to make sense of our experiences, we order them into narratives: sequences of events bound by temporal and implied causal links (Bruner and Lucariello, 1989; Bruner, 1991). As these narratives accumulate in our memory, those that are closest to each other are fused, creating new narratives with less contextual detail and a more general scope. This process is shaped by the discourses in which we engage, by providing both substance and structure. The surrounding culture provides additional experiences – encoded as narratives – with which we fuse our own. At the same time, it provides us with accepted genres for articulating knowledge in different levels of abstraction. Each genre is a system of conversational meta-rules (Bruner, 1991; Sfard, 2007). Thus, knowledge is abstracted from experience. Yet the fine threads from these abstractions to the experiences from which they were derived are never fully severed; these abstractions remain situated in the context of our activities (Noss & Hoyles, 1996).

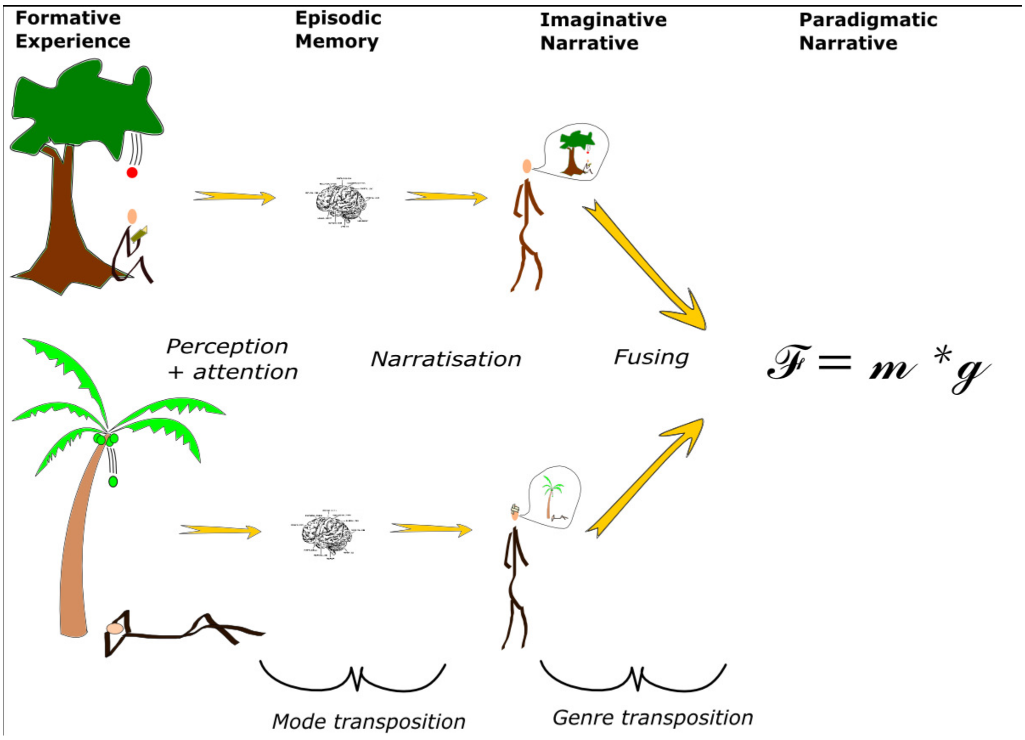

Figure 3: schematic of narrative transposition epistemic model. Experiences are encoded in episodic memory by mode transposition of perceptions. Episodic memory is organised by narratisation, and encoded as imaginative narratives. These are fused in a process of genre transposition into paradigmatic narratives.

Note bene: this is an epistemological claim, not a grand theory of learning. I am not claiming that this is *the* way we construct knowledge, only that it is *a* way, and one worthy of attention. This epistemic model can be read in three lenses: genetic, normative and pragmatic; how we construct knowledge, how knowledge should be constructed, and how we can design for the construction of knowledge. Chapter XX applies the pragmatic lens as a guide for designing tools and activities, and Chapter XX applies the genetic lens to interpret learners' interaction with them. Sections 4.4 and **Error! Reference source not found.** in this chapter focus on the normative perspective, deriving methodological tools and argumentative grammar from the model. Before doing so, the remainder of this section elaborates the base model, and section 4.3 notes some parallels and differences with other approaches.

4.2.1 Narrative and Knowledge

Narrative, in this thesis, is considered in its epistemic capacity. Bruner (1986; 1990; 1991; 1996; Bruner & Lucariello, 1989) identified narrative as the predominant vernacular form of representing and communicating meaning. Narratives are not merely descriptions of *what* happened, they provide an implicit or sometimes explicit explanation of *why* it happened. A narrative, in a nutshell, is an account of *something* happening to *someone* in some *circumstances*. A well-formed narrative must maintain coherence of *temporality* and *causality* (Gergen, 1998). Temporality refers to the chronological ordering of events. The narrative intelligence theory (Mateas & Sengers, 1999), suggests that the identification of temporal affinity of events also plays a strong role in learners' inferences of causality, an important component in the construction of meanings. The sequencing of events is referred to as the *plot*. Gergen (1998) adds that events are carefully selected to support an endpoint. Yet perhaps the most important part of a narrative is typically left unstated: its moral; the narrative's implicit endpoint. A story is told for a purpose – establishing norms, conveying knowledge, or raising a question. It is the implicit layer that holds the narrative together – the causal relationships along the way and the climactic moral at the end. Without them, all we have is an arbitrary list of events. As Mar asserts, "If a well-crafted story contains mention of an event or character, it is assumed that this element is in some way relevant to the goals of the protagonist." (Mar, 2004, p. 1416). To summarise, Narrative is a form of language which includes a context (setting), a protagonist, a plot, and an implicit moral. Narrative also has an affective facet, which includes elements such as voice (the storyteller's presence), and genre (her choice of style and cultural frame of reference). This facet is outside the scope of the current discussion.

Schank and Abelson (1995) argue that stories about one's experiences, and the experiences of others, are the fundamental constituents of human memory, knowledge, and social communication. They call for a shift towards a functional view of knowledge, as Schank (1995) explains: "intelligence is really about understanding what has happened well enough to be able to predict when it may happen again" (p. 1). Such knowledge is constructed by indexing narratives of self and others' experiences, and mapping them to structures already in memory. While Schank and Abelson come from an AI perspective, their theory is supported by recent psychological studies. Atance and O'Neill (2005) define episodic future thinking as the ability to project oneself into the future to pre-experience an event. This, they claim, is a uniquely human phenomenon which precedes semantic future thinking (Atance and Meltzoff, 2005), and provides the developmental basis for skills such as planning and causal reasoning. They found that episodic future thinking emerges around the age of four, and is related to children's abilities to construct and comprehend verbal accounts of experiences.

4.2.2 The Brain as a Narrative Organ

Recent advances in neural psychology ground these observations in new understandings of the brain's inner working (Spreng et al, in press; Mar et al, 2006; Mar, 2004; Holyoak & Krogen, 1995; Young & Saver, 2001; Addis et al, 2004; Mason, 2004). Xu et al (2005) link context to brain regions responsible for global semantic processes such as inference, coherence, conceptual association and text integration. Other findings point to a strong link between narrative comprehension and theory-of-mind processing (Mar, 2004), suggesting that the cognitive modelling of the storyteller and the protagonists is a critical constituent in understanding a story. Taken together, a pattern of learning emerges from these findings. During narrative comprehension, we need to select and then connect the key events of the story. The selection of events utilizes contextual and temporal cues, while connecting them relies on causal inferences. These causal inferences are the genesis of a shift towards a semantic representation of the narrative. In the process, episodic memories are invoked, where they bear resemblance to those in the current narrative. At the same time, causal relationships are retrieved from long-term memory and tested for adequacy. Thus, the perceived narrative is understood as it is woven into a semantic network of past concepts and experiences. As a result, it loses its specificity and becomes itself part of this network, ready to be used as a template to assist in the understanding of future narratives or as a script to predict future events and guide actions. The neural evidence shows that similar mechanisms are invoked in narrative comprehension and construction. It also suggests an embodied element: reading a story which involves actions or physical experiences activates the same regions of the brain that are involved in control or perception of similar experiences (Mar et al, 2006). While some parts of this model have already found a large body of supporting data, others still call for further validation. Yet the picture we see is strongly consistent with existing theories of learning. The invocation of physical experience in narrative comprehension and construction supports an embodied view of learning (Lindblom and Ziemke, 2003; Lakoff and Núñez, 2000; Núñez et al, 1999). In particular, it suggests a mapping of the aspect schema (Lakoff & Núñez, 2000) to the exposition, plot and closure of a narrative.

It is interesting to note the parallel to Vygotsky's theory of learning:

> Although children's use of tools during their preverbal period is comparable to that of apes, as soon as speech and the use of signs are incorporated into any action, the action becomes transformed and organized along entirely new lines (Vygotsky, 1934).

In other words, language plays a critical role in the extraction of meaning from action. Vygotsky talks of *internalizing* knowledge through *inner speech*, and *externalizing* it through spoken words. Typically the subject of a narrative is a protagonist engaged in an attempt to achieve some outcome. The events which she encounters tell us about her actions, but also about the mediating roles of her instruments and her community. In that sense, a narrative is an activity retold.

4.3 The Narrative Turn in Mathematics Education Research

Narrative Enquiry is a well-established paradigm in educational research and teacher training (Conle, 2000; Clandinin, 2007). It is a methodology which "… rests on the epistemological assumption that we as human beings make sense of random experience by the imposition of story structures" (Bell, 2002). Narrative methods are seen as pathways into mathematics education (e.g. Smith, 2006). Much of the work in this vein focuses on narrative as a means of expressing issues of identity and culture, as Bailey (2007:103) sees narrative inquiry as "a journey during which researchers come to know more deeply about their lives and who they are as people" and Kaasila (2007) promotes the value of a narrative view of teacher education in highlighting the personal process of becoming a teacher and construing professional identity. Others also see the notion of *identity* as pivotal: learning is directed by the need to transfer oneself from an actual to a designated identity (Healy and Sinclair, 2007; Sfard and Prusak, 2005). Identities are defined by the stories, narratives, we tell ourselves about ourselves. To understand how learners come to *be* mathematical, how mathematics becomes part of their identity, we need to look at their stories.

This focus on *being* appears at odds with the pragmatist attitude of design science, which forefronts *doing*. Indeed, some authors, such as Markovits and Smith (2008) prefer to use the term cases in order to distance themselves from what they see as "narratives for entertainment". Yet the epistemic model considered above does not preclude knowledge in the pragmatic sense. This model highlights a particular trajectory from experience to knowledge, which involves two phases: the encoding of experience as narrative, and the fusing of narratives through genre transposition. All it requires in order to fit a pragmatist paradigm is that the narratives we choose to inspect are those dealing with change in the world rather than with change in the protagonist's self-perception. I will refer to such narratives as *design narratives* in order to distinguish them from *identity narratives*.

Design narratives are reviewed in the next section. Notwithstanding their fundamental differences, the Design Narrative approach shares a common trait with Narrative Enquiry and case based research. All three attempt to redress a tendency of academic discourse to abstract from practice to a point of detachment from reality. The principle underlying these approaches is the provision of rich and accurate descriptions of reality as experienced by the researcher-practitioner. This principle appears to be very much in line with the pragmatist ideal. However, all approaches risk reducing their discourse to anecdotes. Regardless of whether one considers the plural of anecdote to be data or not,[1] the accumulation of anecdotes does not constitute a theory. As noted above the moral, or conclusion, of a narrative is implicit. Scientific discourse demands that this be made explicit so that it be exposed to scrutiny. Furthermore, a clear method needs to be identified by which knowledge is generalised across individual cases or narratives and captured in a form which can be applied to new situations. The model above suggested an innate process which operates by fusing narratives and genre transposition. Section 4.5 proposes a similar model as a scientific method.

4.4 Design Narratives

As discussed in the previous chapter, design research operates "at the edge of chaos"; research settings and problems are complex, messy and often unique. This creates a challenge in terms of the replicability expected of a scientific experiment. Several authors have noted this difficulty and

[1] "The plural of anecdote is not data" is an idiom popular among sceptics in on-line discussions. An attempt to trace it leads to many references to Frank Kotsonis, although other opinions persist and it is hard to assert a definitive attribution. Several sources note that the original is actually "The plural of anecdote is data", attributed to the Berkeley political scientist Raymond Wolfinger.

proposed the construct of *design narratives* as a means of addressing it (Bell, Hoadley and Linn, 2004; Hoadley 2002; Barab et al, 2008). The main argument in favour of design narratives is that they provide a "thick description" of the design experiment, allowing critics to assess the validity of the researchers' claims, and trace them back to evidence. At the same time, design narratives provide sufficient contextual information for those who wish to conduct a similar experiment in proximal settings, be they fellow researchers or practitioners wishing to apply the research findings.

Design narratives are characterised as accounts of the history and evolution of a design over time, including the research context, the tools and activities designed, and the results of users' interactions with these. They portray the complete path leading to an educational innovation, not just its final form – including failed attempts and the modifications they espoused. Narrative, notes Hoadley (2002:454), "is only one way of making sense of design-based research" but "to really convey what happened, though, requires a story."

Despite the prevalence of the narrative form in reports of design research (Bannan-Ritland, 2003), it raises several methodological and practical issues. In the words of Shavelson et al. (2003:25) "there is nothing in narrative form that guarantees veracity". Practically, narrative accounts do not fit well into academic publication format (Reeves et al, 2005). One apparent source of methodological vagueness is the lack of upfront discussion of the narrative tools used by researchers. With a few notable exceptions (e.g. Barab et al, 2008) most studies intuitively use a narrative style of report without explicitly formulating it as a methodology. The term design narrative itself is rarely used, although many papers are in essence design narratives. Even when the form is discussed, it lacks a rigorous definition: what is the core structure of a design narrative? How are its boundaries set? How are events selected and details filtered out? How should we judge if the narrative warrants the researchers' claims? Section 4.4.1 takes a closer look at these questions.

Another source of difficulty lies within the inherent nature of narrative. In a well-crafted narrative, the message of the story is left implicit (Mor and Noss, 2007). This feature may be epistemically powerful, as it provokes the reader to infer the message and construct her own logical structure to support it. However it is incompatible with scientific discourse, which demands that the path from evidence to arguments to conclusions be exposed to peer scrutiny. The implication is that design narratives are incomplete as a scientific form, and need to be accompanied by a representation of the derived knowledge. Bell, Hoadley and Linn (2004) propose design principles (Kali, Levin-Peled and Dori, 2009); section 4.5 suggests design patterns.

Finally, it is important to remember the interpretive quality of narrative. A narrative is not a neutral recount of events; it is the outcome of the narrator's immediate attempt at making sense of events, a conjecture regarding the semantics of occurrences. Arguably, this is common to all manner of organising evidence: the statistical analysis of a randomised experiment reflects the researchers' choice of parameters and variables. Yet in the case of statistical analysis, another researcher using the same choice of material could have produced the same result. A narrative is unique to its narrator. This subjectivity may be appropriate in design research, where the researcher is part of the phenomena, but nevertheless needs to be accounted for.

4.4.1 Towards a formalisation of design narratives

In order for design narratives to provide an effective form of discourse for design research in education, they need to be shaped in a way that would adhere to scientific standards, acknowledge the agenda of design science, and retain the essential qualities of narrative. This may seem a tall order, but in fact carefully designed forms and procedures for design narratives could allow us to align these forces.

A scientific standard demands a transparent audit trail from reliable data to conclusions, and a clear articulation of refutable claims. Where subjectivity is inevitable, it should be reported honestly. A design science stance dictates a functional (pragmatic) focus linked to a value dimension, attention to context and representation, and an awareness of the complexity of human situations. Narrative form entails a clear context description, a protagonist, a plot – a temporally and semantically linked sequence of events, and an implied moral. Combining these three sets suggest the following set of requirements for design narratives as a scientific instrument:

- Tells the story of an aspect of a design experiment in the voice of the researcher conducting it.
- Clearly delineates the context of the design experiment and its educational goals.
- Presents a documented record of the researchers' actions and their effect.
- Incorporates data collected and processed in appropriate scientific methods.
- Decouples reporting events from their evaluation and reflection.
- Is followed by a statement of the derived conclusions and the warrants linking them back to the narrative.

The conclusion derived from a design narrative is a design claim, i.e. a statement about how to achieve a particular educational effect in a particular context. This claim is external to the design narrative, but it guides the narrator's choice of which events to include in the narrative. Consequently, there can be multiple narratives of the same experiment. All are just as valid, as long as they meet the criteria.

Referring back to section 4.1.1 and Figure 1, the construction of design narratives is a suitable instrument for the interpretation of the raw evidence arising from the empirical actions. The resulting narratives should be useful in themselves, as exemplars for practitioners and peers. However, in terms of the design research process, they need to be processed further in the course of analysis and evaluation. Following the epistemic model presented in section 4.2, this processing can take the form of genre transposition, shifting from the imaginative form of a design narrative to a paradigmatic one. The next section argues for the use of design patterns as a paradigmatic form suitable for the analysis and evaluation of the outcomes captured in the design narratives.

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Design narratives are accounts of critical events from a personal, phenomenological perspective. They focus on design in the sense of problem solving, describing a problem in the chosen domain, the actions taken to resolve it and their unfolding effects. Bruner identifies *Canonicity and breach* as a defining quality of narrative, arguing that "for to be worth telling, a tale must be about how an implicit canonical script has been breached..." (Bruner, 1991, p 11). In the case of design narratives this implies they should either capture a new solution to a known problem, or a new problem. The uniqueness of the single narrative is complimented by its *Accrual* (Bruner, 1991) the manner in which it connects with other narratives to form a coherent body of knowledge.

Bruner (1991) enumerates ten qualities of narrative: Narrative diachronicity, Particularity, Intentional state entailment, Hermeneutic composability, Canonicity and breach, Referentiality, Genericness, Normativeness, Context sensitivity and negotiability and Narrative accrual (Nardi, 2007; Sinclair,Healy and Sales, 2009). *Canonicity and breach* and *Accrual* have been mentioned above as criteria for delineating the whole set of narratives. The others were used as guidelines in the construction of the narratives themselves. These principles required adaptation in order to comply with the norms of scientific discourse, as illustrated in Table 1.

	Bruner (1991)	Adaptation
Narrative diachronicity	"narrative comprises an ensemble of ways of constructing and representing the sequential, diachronic order of human events … its unique pattern of events over time" (p. 6)	
Particularity	A narrative reflects on the generic via the specific. It is an account of an incident, not any incident.	
Intentional state entailment	The actions and events portrayed in a narrative must be relevant to the characters beliefs, desires, theories, values, etc. These cannot be observed directly, yet the story derives its meaning from their induction.	
Hermeneutic composability	The interpretation of a story and the extraction of meaning from it is inseparable from its text, and is part of the implied contract between author and perceiver.	
Referentiality	To be accepted a narrative does not need to be a verified recount of reality, but it must convince the reader that is could have been a recount of reality.	Reference had to be convincingly true to actual events.

Genericness	A narrative is associated with a Genre, which provides a framework for its interpretation	The genre of design narratives emerges from the growing tradition of design research in mathematics education.
Normativeness	The problem in the centre of a narrative illuminates a norm by its resolution or in the absence of resolution by contrast.	
Context sensitivity and negotiability	Assumed background knowledge which modulates the narrative's interpretation and the meaning it implies.	

Table 1: using Bruner's qualities as guidlines for constructing design narratives

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4.5 Design Patterns

The *Design patterns* paradigm (Alexander et al, 1977) was developed as a form of design language within architecture. This was done with the explicit aim of externalizing knowledge to allow the accumulation and generalization of solutions and to allow all members of a community or design group to participate in discussions relating to design. These patterns were organized into coherent systems called *pattern languages* where patterns are related to each other. The use of design patterns never achieved a large following among professional architects, but the idea has been embraced in several other disciplines, starting with software engineering. In their seminal book Gamma et al (1995) argue:

> One thing expert designers know *not* to do is solve every problem from first principles. Rather, they reuse solutions that have worked for them in the past. ... Consequently, you'll find recurring patterns of classes and communicating objects in many object-oriented systems. These patterns solve specific design problems and make ... designs more flexible, elegant, and ultimately reusable. They help designers reuse successful designs by basing new designs on prior experience. A designer who is familiar with such patterns can apply them immediately to design problems without having to rediscover them (Gamma et al, 1995, p1).

Appropriating the ideas of Christopher Alexander, they provided a standard template for software design patterns and taxonomy of 26 patterns. Since then, numerous pattern books, conferences and web sites have proliferated and spread into every aspect of software related design and production. These patterns and pattern languages enable designers to share, discuss and aggregate their knowledge across wide, scattered and diverse communities. More recent examples of areas where collections of design patterns have been created include hypermedia (c.f. German & Cowan 2000), interaction design (c.f. Erickson, 2000 and Borchers, 2001), and mergers of cinema studies and computer science (Walldius 2001). Recently, the concept of design patterns has made its first strides in educational domains. One such domain is that of educationally oriented software systems, such as e-learning systems (Derntl & Motschnig-Pitrik, 2004); another is the design of computer science courses (Bergin, 2000).

The original definition of a design pattern positions it as a high-level specification of a method of design which specifies the context of discussion, the particulars of the problem, and how these can be addressed by the designated design instruments. In *Pattern Languages* Alexander writes:

> Each pattern describes a problem which occurs over and over again in our environment, and then describes the core of the solution to that problem, in such a way that you can use this solution a million times over, without ever doing it the same way twice. (Alexander, 1977)

And in the Timeless Way of Building he elaborates:

> Each pattern is a three-part rule, which expresses a relation between a certain context, a problem, and a solution.
>
> As an element in the world, each pattern is a relationship between a certain context, a certain system of forces which occurs repeatedly in that context, and a certain spatial configuration which allows these forces to resolve themselves.
>
> As an element of language, a pattern is an instruction, which shows how this spatial configuration can be used, over and over again, to resolve the given system of forces, wherever the context makes it relevant.
>
> The pattern is, in short, at the same time a thing which happens in the world, and the rule which tells us how to create that thing, and when we must create it. It is both a process and a thing; both a description of a thing which is alive, and a description of the process which will generate that thing (Alexander, 1979, p 247).

In other words, a pattern has three facets: descriptive, normative, and communicative. It is an analytic form, used to describe design situations and solutions, a meta-design tool, used to highlight key issues and dictate a method of resolving them, and a communicative tool enabling different communities to discuss design issues and solutions. The tension between these three aspects is visible in Alexander's work, and in much of the literature that followed. I will touch upon this issue shortly.

The original collection by Alexander et al (1977; 1979) can arguably be positioned on the *normative* end of the scale, in the sense that an ethical stance can be interpreted from the collection. As Ericksson puts it: "Alexander's Pattern Language is not value neutral" (Ericksson, 2000). On the other hand, Alexander's Mexicalli project is taken as an emblem of participatory design, where patterns are used to facilitate design and empower users – who make their own choices (Dearden et al, 2002). In this case, patterns are predominantly a social tool allowing the expert to communicate knowledge to the families designing their own home. One could claim that there is an ethical agenda here as well. The difference is that in this case it is stated frankly, and given explicitly as the premise – not the conclusion.

Such pattern languages seem to be quite alien to the *descriptive* pattern languages, prevalent in software design. This contrast may stem from Alexander's strong convictions – which need not be shared by many software designers. On the other hand, they may be inherent in the nature of some fields. While in urban planning and architecture it is clear that almost any decision has a political and socio-economic context, it is hard to see such a context in the design of, for example, network routing protocols. However, this distinction should be made with great caution. Design is rarely as value-neutral as we perceive it. The designers' personal, subliminal values are always in the background. Even the software example we used has its ethical dimensions: are the protocols open or closed? Do they allow for encryption? Do they have 'government backdoors'? Such decisions which are often made off-hand have significant consequences in terms of civil liberties. The value dimension of patterns becomes more salient as we move from the core of a technological system (e.g. network protocols, data storage algorithms and database structures) towards the user. Interface and interaction design is laden with such implicit value decisions: does the interface empower the user, or harness her to organizational needs? Is it gender or culturally biased? Does it marginalize users with disabilities? Such questions are generally pushed aside. Perhaps the most notable exception was the Scandinavian participatory design movement in the 1970s, mentioned above, which set forth out of an explicit design agenda of democratizing technology and empowering workers (Kensing & Blomberg, 1998 Asaro, 2000).

I have explored two interrelated dimensions of design patterns: the functional axis (what are they used for) and the value axis. It still remains to mention the *subject* axis – or, what are the patterns *of*? Alexander's patterns are structural – they describe spatial configurations (Alexander, 1977). So are the 'Gang of Four' software design patterns, which describe ensembles of classes in object-oriented programming (Gamma et al, 1995). Other languages aim to design *Actions* (Ericksson, 2000) or *Activity Systems* (Guy, 2004). Digiano et al (2002), for example, interweave three levels in a language of *collaboration design patterns*: whole activity patterns, which describe the dynamics of human interaction, data patterns, which describe the structure and relationships of the artefacts exchanged in the process, and support patterns, detailed patterns which enable higher-order patterns to flow smoothly. The next section discusses the use of pattern languages in educational contexts. Regrettably, it seems that most of the work in this area focuses on structure of digital artefacts and neglects the dynamics of human activity.

4.5.1 Design patterns for learning

As mentioned above, the idea of design patterns originated in architectural theory, but the computer science community was the first to embrace it. It is not surprising that this is also where it has made the greatest impact with respect to education. The design patterns approach has manifested itself through three main trends. The first is the growing trend of *Pedagogical Design Patterns* (Anthony, 1996; Bergin 2000; Eckstein, Bergin & Sharp, 2002). The second is the development of software design patterns for educational technology (Dearden, Finlay, Allgar & Mcmanus, 2002; Avgeriou, Vogiatzis, Tzanavari and Retalis, 2004). The third is the search for patterns in related practices, such as evaluation and assessment (Barre, Chaquet & El-Kechaï, 2005). Nevertheless, it is important to note that the first reference to learning is made by Alexander himself. In his seminal book (Alexander et al, 1977) he describes a pattern called "Network of Learning". The premise of this pattern is that in a society that emphasises teaching, learners become passive and unable to think or act for themselves. He argues that creative and active individuals can only grow up in a society that focuses on learning instead of teaching. The solution he proposes is to replace the structures of compulsory schooling in a fixed place, with decentralised processes of learning which engage learners through contact with many situations and people all over the city: workshops, teachers at home, professionals willing to take on the young as helpers, older children, museums, youth groups, scholarly seminars, industrial workshops, old people, and so on. This argument resonates with Illich's call for “deschooling society” (1971) and conviviality (1973). We find such arguments motivating in our search for alternative means of mathematical learning.

Pedagogical design patterns apply the concept of design patterns to pedagogical design. The fundamental claim behind this effort is that many experienced practitioners in education have tried and tested methods of solving recurring problems or addressing common needs. Among the pioneers in this field were Anthony (1995) and later the pedagogical patterns project (http://www.pedagogicalpatterns.org/), initiated by a group of experienced software engineering and computer science educators (Bergin, 2000; Eckstein, Bergin & Sharp, 2002). They proposed a set of patterns dealing with issues ranging from the design of a college course to specific principles of computer science instruction and to concrete problems and their solutions.

A second arena that has seen a proliferation of design patterns over the last years is web-based educational technologies. Notable examples in this field include the E-LEN project (http://www2.tisip.no/E-LEN/) and several initiatives within the IMS-LD framework (http://www.imsglobal.org). Most of the work in this area is focused on the engineering aspects of designing, developing, deploying and evaluating good technology for web-based instruction (Frizell & Hubscher, 2002; Hernández-Leo et al, 2006; Bailey et al, 2006)

This strain of work is done mainly in the context of developing large scale technological systems to support organizational and vocational learning or web-delivered higher and further education. Due to this context, much of the work is highly technical. Many of the valuable innovations have a strong engineering flavour to them (e.g. Bailey et al, 2006) which might deter teachers and educational researchers. The interaction between student and instructor is assumed to be mediated exclusively by web-based communication channels. Under such circumstances, most of the effort goes into designing the representation and organization of educational content and the mechanisms by which learners interact with it (Frizell & Hubscher, 2002). Design patterns are also situated in this context, with the engineer of educational technologies as the user in mind (Avgeriou et al, 2003; Garzotto et al, 2004; Kolås & Staupe, 2004). From this perspective, pedagogical issues are often assumed rather than discussed. A noteworthy exception is Goodyear (2004). In an attempt to distance himself from the dominant approaches in e-learning, Goodyear focuses on what he calls networked learning, where technology is used to promote connections between learners and foster communities which make efficient use of their resources. In this context, Goodyear emphasises patterns as a means of empowering practitioners to utilize accumulated design knowledge. His patterns are succinct and written in plain language. Another study oriented towards educators is Dearden at al (2002; 2002b). They point to the strong ideological and methodological parallels between Alexander's original vision of pattern language and the paradigm of participatory design. They propose the 'facilitation' model developed by Alexander et al (1985) in the Mexicali project as an alternative to the dominant approach of using patterns to deliver expert knowledge to novices.

Finally, design patterns have recently been used in the context of assessment, evaluation and analysis of learning and learning systems. Pachler et al (2009) apply a pattern-based methodology in the domain of formative e-assessment. Wei, Mislevy, and Kanal (2008) provide an extensive collection of patterns for language assessment. Gibert-Darras et al, (2005) offer a pattern language for assessing students’ problem solving abilities in the context of a basic Java course. The standard

Alexandrian argument holds here as well: assessing students' performance is a hard job, into which a lot of research has been done and many practitioners have accumulated insights through experience. Patterns allow us to offer this knowledge in a useful form to novice teachers.

To conclude, with the exception of Dearden et al (2002a; 2002b), Goodyear (2004) and Bergin et al (2000, 2002), most studies which utilize design patterns in education are concerned with the hard issues of creating good educational technology and authoring content within technological systems. Without downplaying the importance of such endeavours, I see a potential for pattern languages which refer both to the technological and the pedagogical aspects of designing environments and opportunities for learning. They should elucidate grounded principles of design of content, activities and tools, and the relationships between them. To achieve this, they would need to be integrated with a range of tools and representations for capturing design knowledge (.

Another uncharted territory it the pedagogical dimension of known software design patterns. For example, the *Streams* pattern, presented in section XX, is widely used in application and systems programming. When approached from an epistemic-pedagogical perspective, it has other qualities of note such as its correspondence with intuitive notions of sequences and infinity (section XX). Another example is the *model-view-controller,* or *MVC,* pattern (Krasner & Pope, 1988; Gamma et al, 1995). This pattern separates the representation and manipulation of information from its structure and content. MVC is perhaps one of the most powerful and widely used patterns in interface design. From a pedagogical perspective, it resonates well with the discussion of representations (Balacheff & Kaput, 1966; Radford, 2000). Indeed, this pattern is utilized in the design of ToonTalk (Kahn, 1996). However, most educators involved in constructionist activity design are not aware of it. Furthermore, the pattern's common descriptions are focused on the engineering aspects and do not expose its epistemic qualities. For educational designers to leverage the benefits of this pattern, it needs to be expanded in a manner that will bring together both worlds, that of software engineering and that of educational design.

4.5.2 Design patterns as expert situated abstractions

Salingaros (2000) argues for the timelessness of Alexander's pattern language. Yet if we see the design, production and usage of tools as a dynamic process of social learning, timelessness is lost. In the iterations of a design experiment, patterns are constantly shifting and reforming. Even the act of eliciting and formalizing a pattern may lead to its own evolution, as it is adopted and modified by the community. Paradoxically, the more successful a pattern language is, the less stable it would be. In part, this discrepancy with Alexander's theory can be attributed to the nature of digital technology. The design of urban environments is constrained by characteristics of the physics and geography of our world, the structure of the human body and the workings of the brain, all of which are slow to change. Hence, most of the patterns that Alexander identified in the 1970s are just as relevant today as they probably were 200 years earlier. In the case of digital artefacts, the extent of human imagination is often the only constraining factor. A second difference is the breadth and depth of design activity. When building a house I can design its layout, but I must choose among available tools and materials. With digital artefacts, this distinction is blurred. Furthermore, digital technologies give widespread access to domains of design which were previously laborious and professionalized.

The observations of the previous section suggest a change of emphasis on the role of design patterns. Alexander's patterns may have aimed at capturing age-old design knowledge and make it available to a wide audience. In the domain where age-old knowledge does not exist, patterns should instead aim at identifying elements of effective practice as they emerge, and capture them as objects for discussion, scrutiny and manipulation. Alexander's architectural design patterns are informed by theories of construction, engineering and human psychology. In much the same way, pedagogical patterns should be based on a theoretical layer concerning pedagogy and epistemology. Whereas, for example, a software design pattern may need to include justification in terms of computational efficiency and robustness, a pedagogical design pattern should include its epistemological, psychological or social dynamic rationale. Unfortunately, this is rarely the case. Since most pedagogical patterns are developed by skilled practitioners (or software engineers) who have their formal grounding in computer science rather than educational sciences, they are rich in solid intuitions but much less informed in educational theory.

Furthermore, design patterns could offer a potent analytical tool for design-based study of technology-enhanced mathematical learning. As observed by Chronis Kynigos (personal communication), design patterns are the researchers' situated abstractions. Design patterns offer a method for gradual abstraction out of personal experiences. This method is particularly relevant in the design-based research context, where personal reflection on design experiments is considered valid data. Design patterns retain critical elements of narrative form, and add structure which allows us to see links, hierarchies and systemic forces.

4.5.3 The Promise of Design Patterns

Section 4.4 suggested a need for semi-formal notation to be used in conjunction with design narratives and help capture the design knowledge derived from them. Several forms have been suggested for capturing abstractions of design knowledge in education, among them design principles (Kali, Spitulnik and Linn, 2004; Kali and Ronen, 2005; Kali, 2008; 2005; Kali, Levin-Peled and Dori, 2009), scripts (Miao et al., 2005; Kobbe et at., 2007) and sequences (Dalziel, 2006). McAndrew, Goodyear and Dalziel (2006) compare a few of these. This section considers the qualities of the design pattern form which make it a suitable candidate for complementing design narratives as a component in an epistemic infrastructure for a design science of education.

The core of a design pattern can be seen as a local functional statement: "for problem P, under circumstances C, solution S has been known to work". Such as structure reads like a direct generalisation of the narrative form of "something happened to someone under some circumstances", when that narrative is a record of a problem solving effort – in other words, a design narrative.

By forefronting the problem, the structure of a design pattern acknowledges the functional axis of decomposition and the value dimension, identified in Chapter XX as tenets of design science emerging from Herbert Simon's work. These features are further expressed in the links between patterns, inherent to the pattern format. Christopher Alexander (1999) explicitly highlights what he calls the "moral" and "generic" qualities of pattern languages, and asks whether these are present in the way the idea has been appropriated by computer science.

Complexity and context-dependence are characteristic of design based research in education which emerge from the discussion in Chapter XX. The design patterns approach is sensitive to these issues, and reflects them by restricting solution statements to compact classes of problems in clearly delineated contexts. In this sense, a design pattern can be seen as a representation of a local theory or a modular ontological innovation, to use diSessa and Cobb's phrase (2004).

The modest nature of design patterns can also be seen as an expression of a pragmatist philosophy, suggested by several authors as the foundation of design-based research. This philosophy supports the notion of ontological innovations, which diSessa and Cobb (2004) derive from the need to address the gap between practice and theory. Design patterns were described as situated abstractions of expert knowledge; they generalise from successful practice without detaching from its context. As such, they offer a two-way bridge between practice and theory: opening practical wisdom to theoretical scrutiny and allowing theory to be projected into practice. A pragmatist perspective leads many design researchers to seek holistic frameworks, calling on diverse mixes of theories and methodologies in the service of comprehensive solutions. The core structure of design patterns is conducive to such an approach, as it demands precision in description of problem, context and solution, and subjects to them theory and evidence.

The functional, holistic, compact form of design patterns also makes them promising candidates to serve as boundary objects in design-level interdisciplinary discussions. Following Bowker and Star (1999), there is a growing acknowledgement that practitioners from different communities interfacing in a joint enterprise may inhibit distinct activity systems (Tuomi-Gröhn and Engeström, 2003). Consequently, the conceptual spaces that these communities form around the joint enterprise would diverge, impeding communication and coordinated resolution of emerging issues. Boundary objects are artefacts that might help to calibrate the diverse perspectives towards a shared canon of knowledge situated in common problems (Noss et al., 2007; Bakker et al. 2006). As argued in Chapter XXX (and in Mor & Winters, 2007), educational design is an inherently multi-disciplinary activity. An effective study of design – whether scientific or practical – demands linguistic and symbolic tools which will enable boundary crossing and facilitate discussion between the various vested communities, to ensure that solutions and analysis take all critical factors into account. Design patterns – if carefully crafted as products of interdisciplinary discussion – may emerge as such boundary objects.

Finally, design patterns have been used extensively in object-oriented programming for over a decade. Apart from their popularity amongst software designers, recent studies indicate measurable benefits in terms of cognitive load (Kolfschoten et al., 2006), software quality (Guéhéneuc et al., 2006) and system maintenance (Prechelt et al. 2001). Evaluating the effect of design patterns is neither trivial nor conclusive; as noted by Khomh and Gueheneuc (2008) they can also have negative consequences. Furthermore, it would be irresponsible to suggest a simple analogy from software development to education. Nevertheless, such results do raise the possibility of added value for both educational design as a practice and its scientific study.

4.5.4 Challenges for the Design Pattern Approach

Section 4.5.3 argued for the potential of design patterns as components in an epistemic infrastructure for design research in mathematics education. Design patterns, introduced by

Alexander in 1977, have been appropriated for education more than a decade ago (Eckstein, Manns and Voelter, 2001). Yet so far they have not witnessed wide-spread adoption in educational practice or research. To an extent, this could be attributed to sociological factors: for many years, the design pattern approach was predominantly visible only within the community of architects. The impact of the approach in the world of software design was only noticeable after the publication of Gamma et al.'s book in 1995. Design patterns entered the world of education through the "back door" of educational technology and computer science education. However, in education the design patterns approach does face some fundamental challenges which impede its acceptance. Notably, it raises issues of validity, resonance, cumulativity and innovation.

Validity refers to the scientific confidence which can be attributed to a pattern. Much of the pattern literature positions itself as expert insights rather than a component of scientific research. Thus, common patterns are supported by intuitive arguments and personal conviction derived from professional experience. Christopher Alexander himself declares "I am a scientist" (2008), but does not present the scientific method by which his patterns were derived. In order to be accepted as part of a scientific discourse, patterns need to include a clear audit trail linking them to data, and incorporate reference to relevant theoretical warrants. They also need to be situated within and in relation to other forms of scientific practice.

The issue of resonance relates to the impact design patterns have in the relevant communities. This is something of a "chicken and egg" issue: as a linguistic form, a tool for conversation within and across communities, the more widespread patterns are, the more useful they become. Yet without a critical mass acknowledging their usefulness, they remain esoteric and marginal. This problem is compounded by the "cultish" nature of some pattern communities, which do not appear to be concerned with making their discourse accessible to others. Indeed, while a substantial literature of pedagogical patterns appears to be building up, there is a shortage in introductory texts, initiating new audiences into the art of reading, writing and using patterns. More crucially, one of the core virtues of design patterns is an obstacle to their adoption; as noted above, design patterns inhibit a space between theory and practice. This allows them potentially to bridge between the two, but it also raises the risk of being perceived by practitioners as too abstract and by theoreticians as trivial. In order to overcome these obstacles, design patterns need to strengthen their link both to practical examples (e.g. through associated design narratives) and to theoretical warrants and consequences. Design patterns need to be embedded within a wider scientific and practical discourse, and their relationship with other formats and methods established.

The issue of cumulativity concerns the extent to which existing knowledge is used as a foundation for new developments. By definition, design patterns describe the common elements of recurring solutions to recurring problems. It would seem that knowledge aggregation is inherent in patterns, as they attempt to provide a shareable, reusable representation of expert knowledge. Yet the literature is dominated by collections of novel patterns, while reference to prior collections or evidence of their use is sparse. In computer science, the commercial proliferation of pattern language books (e.g. Wiley publishers software design patterns series) suggests that at least the knowledge captured by these languages is reused, if not cumulated. Unfortunately, when it comes to educational design patterns the evidence of use is inconclusive at most. Several factors might possibly contribute to the lack of cumulativity. The lack of cumulativity may be related to a common positioning of pattern writing as a "discovery" process, rather than one of "construction". A prevalent view assumes that patterns are “out there” in the world, and the task of the pattern author is to uncover them. This view ignores the fact that for many authors of pattern, the process of identifying and articulating a pattern is in itself a trajectory of learning, interpretation, and construction of personal local theories. In such cases, the author might be more interested in expressing her own patterns than in using others’. Scientific cumulativity is a property of a community, and needs to be embedded in its culture. In order to achieve this property, design patterns need to become part of an on-going discourse in a design science community attuned to its cycles of enquiry.

The issue of cumulativity, and in particular the observation of pattern-writing as a trajectory of learning, leads to the tension between effectiveness and validity on one hand and innovation on the other. This tension is inherent in design research (Schwartz, Chang and Martin, 2008) and is amplified by the original positioning of design patterns as a record of established practice. As the title of “The Timeless Way of Building” (Alexander, 1979) suggests, Christopher Alexander saw design patterns as capturing timeless qualities of architectural solutions which have been observed consistently across time and location. By contrast, design research in education deals with innovation – identifying novel solutions, and using these as tests for educational theories. This is even more the case when dealing with technological elements, where the problems themselves are fluid and the context constantly shifting. The tension between established “good practice” and innovation echoes the tension between the dual facets of design science, advancing theory on the one hand and supporting practice on the other. Alexander himself may hint at a way to reconcile these forces. In *A pattern language* (Alexander, 1977) he assigns three degrees of confidence to different patterns. Borchers (2001) notes that such ranking helps the community distinguish and position each pattern on the scale between the universal and the experimental. Some patterns aim to highlight truly timeless solutions, others to provoke questions and new perspectives by proposing radical changes. A good pattern language should strive to balance these, clearly identifying each pattern on this scale. Furthermore, articulating reputable solutions as patterns has its own value for innovation, as it allows us to scrutinize and refine them, and to transfer them to new settings. Educational and technological innovations often focus on one aspect of a problem situation, succumbing to naïve assumptions along other dimensions. Design patterns allow us to build innovations on solid foundations, containing their vulnerability to the conjecture being tested. When combined with suitable complementing representations (such as design narratives) and supported by meticulous research practices, design patterns hold a potential for aligning cumulativity, effectiveness and innovation, while balancing practical value and scientific validity.

4.6 Bringing it all Together: Attaching the Representations to the Research Cycle

Section 4.1.1 presented a cycle of design experiments in education, and identified a need for representations of knowledge appropriate for each phase, and the means of transitioning between them. Section 4.2 reviewed an innate process of extracting knowledge from experience, as a possible analogy for such representations and transitional mechanisms. Using such an analogous model has two advantages. First, it lends credibility to the epistemic process by grounding it in a parallel that is known to work, while reinforcing it by formal structures which add scientific rigor. Futhermore, it enhances the readability of the scientific process, by drawing on familiar dynamics, thus making it more transparent to academic scrutiny and more accessible to practitioners.

Section 4.4 introduced design narratives and section 4.5 complemented them with design patterns. The latter were presented as representations of design knowledge with a growing degree of abstraction. Using the innate learning model as a guideline, it is possible to set these representations into the design experiment cycle and see the transitions between them as genre transpositions. Figure 4 illustrates one possible embedding; the events of the implementation and action phases are captured by design narratives, augmented with any data that can support them. Design narratives are used as a basis for pattern extraction, producing initial design patterns. These patterns are substantiated by reference to theory and additional supporting evidence. The resulting mature patterns are offered as outputs to the community, and at the same time fed back into the design and implementation phases in preparation for the next iteration of empirical work.

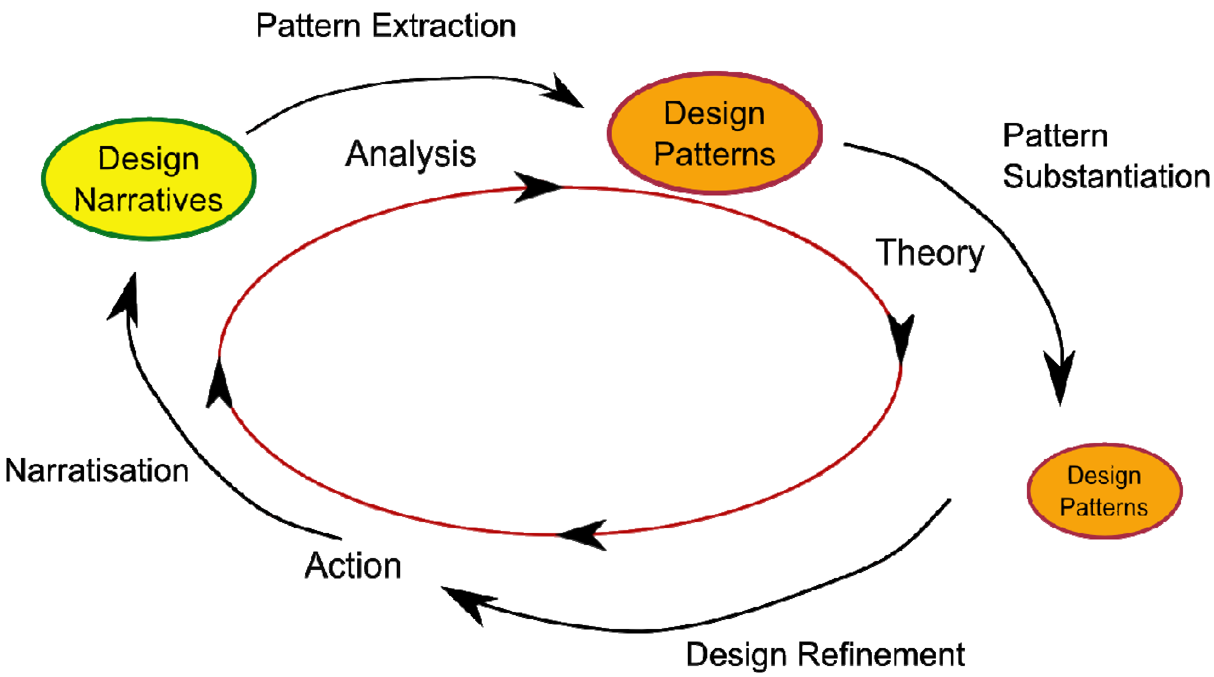

Figure 4: design narratives and design patterns set in the design experiment cycle

4.7 Methods

The coming of age of the design based research (DBR) paradigm in education is marked by emerging methodological standards (Cobb et al, 2003; Shavelson et al, 2003) which are characterised as iterative, process-focused, interventionist, collaborative, multileveled, utility oriented, and theory driven. The applicable methods of data collection and analysis are derived from the nature of research. Controlled variables and pre- and post- tests give way to intensive process-oriented observations. A mixture of mainly qualitative methods is used, including video and audio recording of student activities, analysis of texts and artefacts produced in the course of these activities, interviews and ethnographic field notes. The collaborative nature of DBR poses an apparent challenge to doctoral research, given the requirement to demonstrate individual contribution. This tension can be negotiated by defining clear responsibilities and keeping an account of the process, e.g. as a research journal.

Drawing parallels between DBR and other methodological paradigms provides a guideline for evaluating specific research tools. DBR shares the situatedness of *action research*. Similar to grounded theory, it strives to make theory relevant by maintaining a strong link to practical experience. It resonates with the subjective stance and meticulousness of phenomenology. Juuti and Lavonen (2006) compare DBR to other paradigms, such as didactical engineering (Douady, 1985; Artigue, 1994), noting that such approaches aim to produce theory-informed educational design, whereas DBR is equally concerned with theoretical innovation. What sets DBR apart from these other paradigms, particularly in the case of educational research, is, perhaps, its fundamental interest in questions of epistemology. The situation which is studied is not the focus of attention: it is a window into participants' experiences and through them to common human traits. Putting human experience in the centre, while acknowledging the subjectivity of the participant-observer, links DBR with the phenomenological approach. Yet phenomenology sees the detailed description of the personal experience as its main concern (Denscombe, 2003). For DBR it is a step on the way to modest and careful theoretical claims. These derive further support from triangulation with other empirical results, often from diverse fields of research.

Mapping the links between design-based educational research and other paradigms suggests that some of the specific tools used by these traditions may be relevant to the present study. Yet there are two unique characteristics of DBR which would require careful adaptation of any tool. First, the focus of inquiry in DBR is human learning and how it is affected by educational design. Obviously, any research tool used would have to be calibrated towards this perspective. Second, DBR is distinguished by a tight, dynamic and continuous interaction of design, experimentation and analysis. In contrast with other approaches, it is not unusual for data collected in one week to be quickly analysed and used to redesign the session for the following week. This requires agile methods of collection and analysis which can fit into such an intensive cycle.

Data is analysed at several levels of granularity. The microgenetic level (Brown, 1992) offers immediate and specific indications regarding the relations between details of design and incidents of learning. These serve as input for the dynamic adjustment of designs, and at the same time for the accumulation of data to support higher-level theoretical statements. Intermediate analysis takes place between iterations, revisiting the design of tools and activities in light of the evidence collected during and after the experiment. This level of analysis supports adjustments to design within the existing structure. Such adjustments allow researchers to reduce 'noise' by removing superfluous usability flaws, and at the same time test conjectures which emerge from the data by designing elements which relate specifically to predictions derived from them.

Reflective analysis is a critical phase of a design study, which takes in the totality of microgenetic and longitudinal observations made over several iterations, and reflects on them in light of existing theory to produce new fundamental claims. The complexity arising from the real-life nature of the research settings, along with the highly interventionist method of study, call for detailed personal longitudinal accounts of design and experimentation processes. Such accounts, argue Shevelson et al (2003), tend to take a narrative form.

4.7.1 Observation-in-action

Augmenting the principles of DBR with the ideas of situated abstraction (**Error! Reference source not found.**) suggests an emphasis on thinking-in-change (Noss & Hoyles, 1996) within an activity system. The notion of thinking-in-change emerges from the view that knowledge is not transferred instantaneously from teacher to learner, but is dynamically constructed through constructive activity and reflective communication. This implies that in order to study learning, we need to observe the process by which it proceeds: pre and post comparison does not suffice. It follows that the instruments used should be capable of capturing incidents in which knowledge germinates or coagulates as they occur. Pratt (1998) argues that this leads to a focus on description and interpretation, in the tradition of ethnography or anthropology. Yet as he notes elsewhere (Pratt et al, 2006) the interventionalist nature of DBR leads to a methodology based on perturbing thinking (Noss and Hoyles, 2006): confronting the learner with new situations, provoking her to act, and observing how her thinking changes in response. The ideal of the participant-observer is extended to a principle of observation-in-action: the researcher disrupts the learner's experience, provokes her to act or express herself verbally, and tries to capture the minute indications of thinking-in-change as they occur. This principle applies to all levels of granularity, from the microgenetic to the reflective.

It is important to acknowledge the absence of 'pure' observations in a DBR setting. There are no 'one way mirrors'. When the observer is a key participant, any action he takes in the face of learners is an intervention. Taking note of a learner's casual articulation lends it significance in that learner's mind. Rather than trying to ignore this factor, it is better to incorporate it into the design and practice of research, while accepting the implied limitations. Instead of pretending to make unobtrusive observations, it is better to say: 'that's interesting. Let me take note of that. Can you explain?' Inevitably, such an approach blurs the distinction between observation, interview and intervention. Special care is required in order to collect robust observations while minimizing the disruption to the learning process. This balance is achieved by combining a multitude of observation methods, some passive (e.g. video), some active (e.g. in-activity probes) and some post-event (e.g. stimulated recall interviews). These methods are described in the remainder of this section.

4.8 Conclusions

This chapter started by highlighting some methodological challenges facing a design science of technology enhanced mathematics education. These challenges all reflect the overarching need for a clearly articulated consensual epistemic infrastructure.

As a first contribution towards the formulation of such an infrastructure, two concentric cycles of scientific process where presented. These cycles were abstracted from the reports of various research groups regarding their practices.

While these cycles plot a general framework for conducting research, they need to be realised by providing appropriate representations for expressing design knowledge in their various phases, along with procedures for managing the transitions between phases. A particular innate trajectory from experience to knowledge was reviewed as a basis for these representations and transitions. This mechanism proceeds by narratisation of experience and genre transposition. Genre transposition fuses similar narratives, thus abstracting similarities and eliminating detail. In this process temporal relations are replaced by semantic ones.

By analogy to this model of innate learning process, two representations were proposed: design narratives and design patterns. The former serves the interpretive phase of the design experiment cycle, in which the researcher organises the data and records the unfolding of events in the empirical phase. The latter serves the analytical and conjectural phases, allowing researchers to articulate situated abstractions of design knowledge derived from the experiment.

Several issues were identified with the intuitive notion of narrative, leading to a proposal for a formalisation of design narrative as a form of scientific discourse. Design patterns emerged as a promising form for encoding design knowledge in educational research, but likewise, several challenges were identified. These challenges can be met by providing a rigorous methodological apparatus, which would include the detailed format of each representation and the procedures for transitioning between them, along with measurable criteria for validity. Such a framework will be the subject of Chapter Five.