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Chapter 5 Methodological Framework, Methods and Settings

This chapter addresses Aim 2 in Chapter Three, by describing the methodological framework and the set of methods used in the demonstrator study defined by Aim 3. The framework and methods are derived by projecting the approach detailed in Chapters Two and Four onto the specific research settings. These settings are specified in this chapter as well.

5.1 Introduction: a Methodological Framework for a Design Study of Learning about Number Sequences

The primary research question of this thesis, as expressed in Chapter Three, Aims 1 and 2, concerns the formulation of methodological tools for design research in technology enhanced mathematics education. This question was addressed in Chapters Two and Four. Chapter Two argued for a design-based approach to research in technology enhanced mathematics education (TEME). Chapter Four extended this argument, to propose elements of an epistemic infrastructure for design research in this field. In order to assess the validity and utility of these elements, they need to be applied to a concrete, genuine and non-trivial research problem. Such a demonstrator problem is defined by Aim 3 in Chapter Three: "the design of tools and activities for learning about number sequences, in an extra-curricular lower-secondary school setting". Chapters Six, Seven and Eight communicate a study of this problem in the context of secondary school extra-curricular activities with number sequences.

The current chapter operationlises the elements and provides the methodological framework along with a set of suitable research methods, with respect of the given questions and settings of the demonstrator study. The primary constructs presented in Chapter Four were the two cycles of design research, design narratives and design patterns. This chapter describes the research setting chosen to illustrate these constructs. It then explains how they interacted in the context of this setting, and identifies the processes by which design narratives and design patterns were produced and validated.

The demonstrator study was conducted between September 2002 and December 2006, with some refinement of analysis continuing beyond that date. The work as a whole followed the design research meta-cycle (Figure 1) described in Chapter Four.

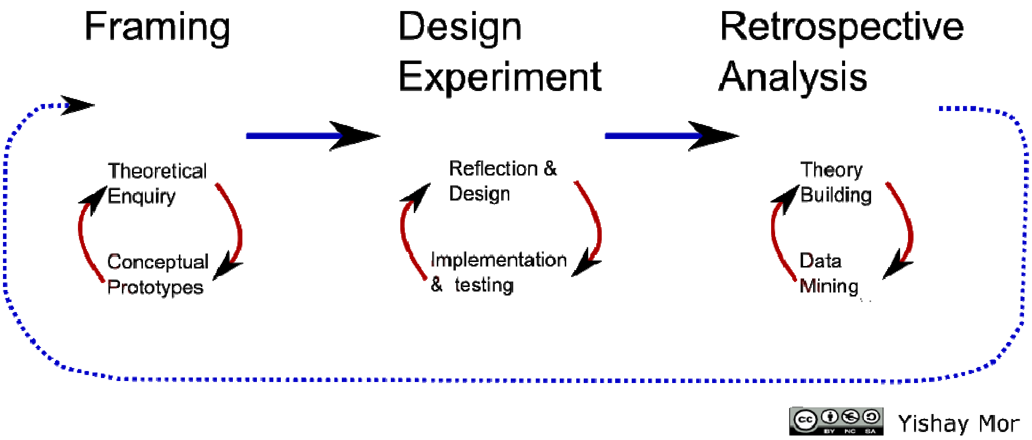

Figure 1: design research meta-cycle, presented in Chapter Four, reproduced for convenience

The framing phase of the research occurred between September and December 2002. This phase included a scoping study, construction of conceptual prototypes in ToonTalk and an initial outline of activity and tool designs. The scoping study provided the basis for the review presented in Chapter six. However, this review evolved in tandem with the design throughout the following phases.

The empirical design experiments proceeded from December 2002 to June 2005. They involved three major iterations and several intermediate adjustments. The first iteration was a loosely designed pilot study and was dominated by a trial and error strategy. The final iteration enjoyed a stable design, shifting the focus to data collection. Consequently, most of my comments on the process of design refer to the first two iterations, while the epistemic observations rely mainly on evidence from the third year. By and large, these experiments followed the design experiment cycle (Figure 2) discussed in Chapter Four. Each new iteration was motivated by reflections on the previous one, were the first iteration was driven by reflections on the prototyping from the framing phase. These reflections were supported by data collected and interpreted in a variety of methods, as detailed in section 5.3. The empirical observations where supplemented by revisiting the literature to provide a sound basis for the next revision of design.

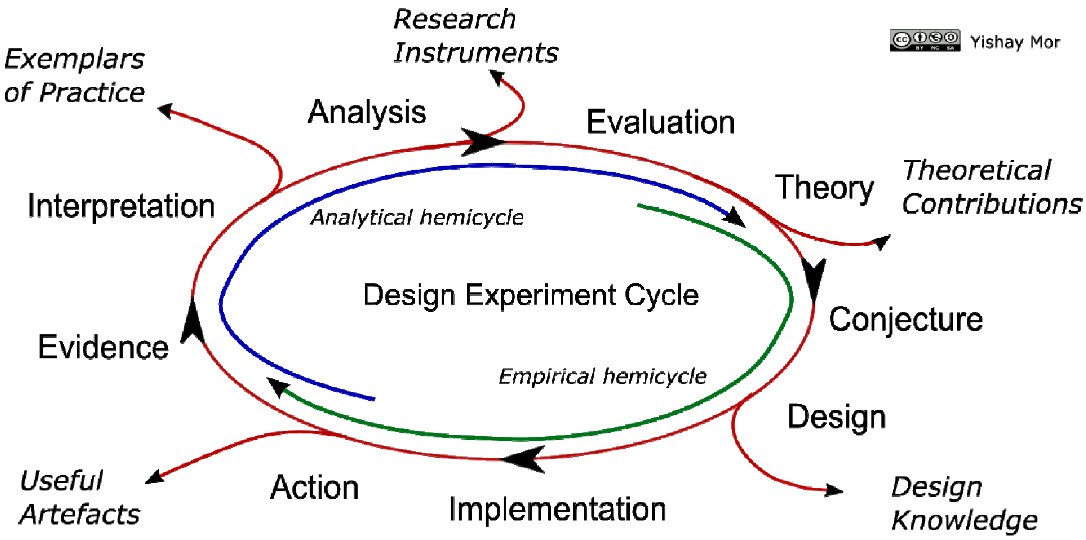

Figure 2: Design experiment cycle, presented in Chapter Four, reproduced for convenience

The three design iterations were followed by a phase of reflective analysis, the main bulk of which was performed between July 2005 and December 2006. In this phase, I took a step back to scrutinize my study as a whole, distancing myself from the specific details of classroom incidents to take note of general themes and meta-questions. It is at this stage that design narratives and patterns came to play. Design narratives allowed me to consolidate observations within and across the three iterations, and promoted a systematic interpretation of the data collected in the experimental cycles. The design patterns derived from these narratives highlight the effective methods and tools which emerge from my reflections across the three years of empirical work. These patterns point at key issues and obstacles in the chosen domain of knowledge, and suggest possible ways to address them. Patterns are grounded in theoretical arguments and validated by a range of qualitative methods. Section 5.4 presents the processes by which design narratives were chosen and developed. Section 5.5 notes how design patterns derived from the design narratives and substantiated.

5.2 Experimental Setting

The context of the demonstrator study was defined by the *WebLabs* project (www.weblabs.eu.com, European Union, Grant # IST-2001-32200) directed by Professors Richard Noss and Celia Hoyles. WebLabs provided both the research setting and the technological infrastructure for my study. WebLabs aimed to explore new ways of constructing and expressing mathematical and scientific knowledge in communities of young learners. The project's approach brought together two traditions: *constructionist learning* as described by Papert & Harel (1991) and collaborative *knowledge-building* in the spirit of Scardamalia & Bereiter (1994).

5.2.1 Classroom setting

The empirical part of this study was conducted in two locations in London over three consecutive years, from 2002 to 2005. The experiments involved several groups, as detailed in Table 1. All groups were instructed and observed by Gordon Simpson and myself. The collaborative activities involved interaction with other sites in Oxford, Sofia, Nicosia and Lisbon. Most of the activities were also tested and evaluated independently in each of these sites. The analysis in this study focuses on published materials from the London groups, with reference to other sites when relevant. Interviews and observations were collected from the London sites alone.

It.[1]	Gr.	Period	Location	F	M	Age	Sessions
1	I	Autumn 2002	After school club, central London.	3	3	11	10 weekly, 90 minutes, + full day workshop.
2	II	Autumn 2003	Lunchtime club, central London.		6	10-11	10 weekly, 50 minutes, + full day workshop.
2	III	Autumn 2003	In lieu of ICT class,	4	4	13	10 weekly, 50 minutes, + full

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[1] See section 5.2.3 for a discussion of the nature of the different iterations.

			central London.				day workshop.
3	IV	Autumn 2004 & Spring 2005	After school club, north London.	10		13-14	16 weekly, 90 minutes, + 2 full day workshops.
3	V	9 June 2005		2	3	12-13	One full-day workshop (previous sessions with Ken Kahn).

Table 1: Experiment Groups

5.2.2 Technological setting

The technological approach of the WebLabs project consisted of two tightly related components: a programming environment for students to construct models of their ideas and a web-based collaborative environment for them to share them. ToonTalk (www.toontalk.com) was chosen as the programming platform, while the WebReports collaborative system was designed by me and developed by our team in the course of the project. The technological platform used by the WebLabs projects manifests a particular educational approach. The nature of the project was such that the underlying pedagogy and the supporting technology shaped and reshaped one another. The initial configuration of the infrastructure reflected the initial pedagogical conception. After the first round of experiments, the concept was adjusted to accommodate the lessons learnt regarding the potentials and limitations of the platform, and in turn the platform was reconfigured to adapt to the pedagogical change. My personal contribution varied across this cycle: I provided feedback on the development of the ToonTalk environment, participated as an active team member in the development of the educational framework, and led the design and implementation of the WebReports system.

Section 5.2.2.1 provides a brief overview of ToonTalk, and 5.2.2.2 notes the fundamental features of WebReports, as they pertain to the data collection and analysis.

5.2.2.1 The ToonTalk programming environment

ToonTalk is a language and a programming environment designed to be accessible by children from a wide range of ages, without compromising computational and expressive power (Kahn, 1996; 1999). It does this by embedding complex programming constructs in a video-game setting as shown in Figure 3. In ToonTalk, every programming structure is concretised as an animated cartoon object: robots (labelled 2 in Figure 3) stand for programs, boxes (labelled 3) for data structures, birds (5) for message sending, nests (6) for message receiving, scales for comparisons, trucks for process spawning, and bombs for process termination. The toolbox (11) contains the data types and operators, while the notebook (12) provides a standard library of stored procedures.

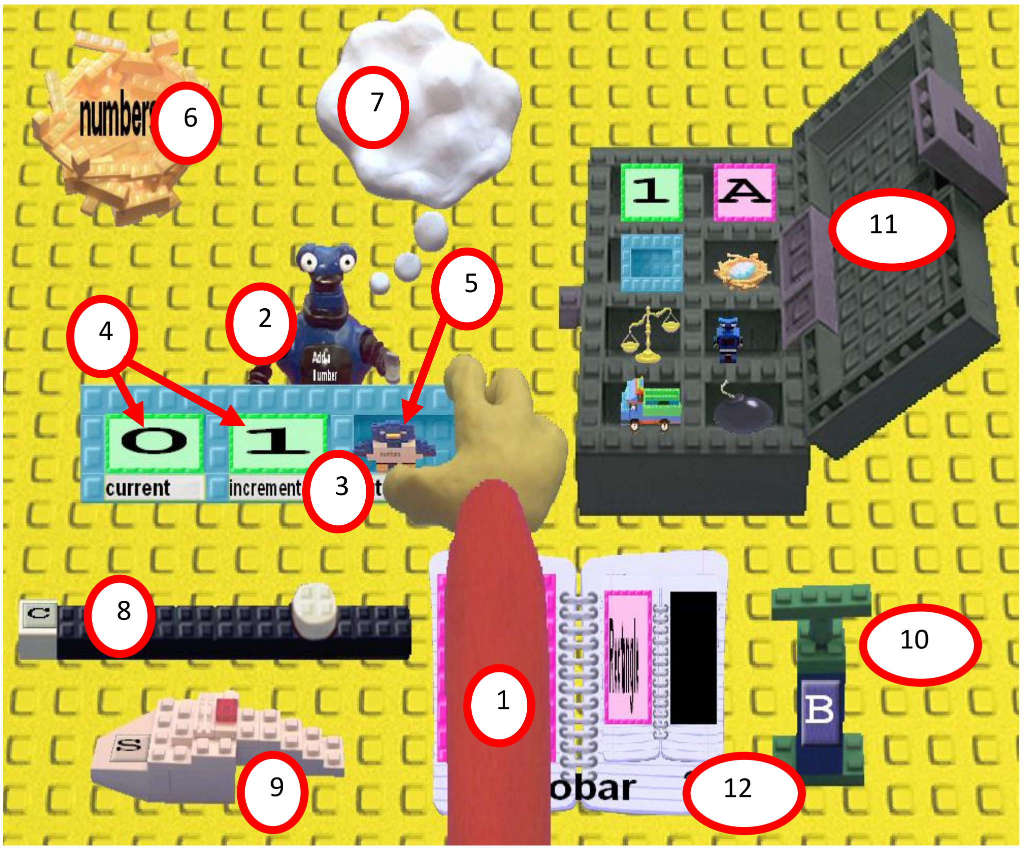
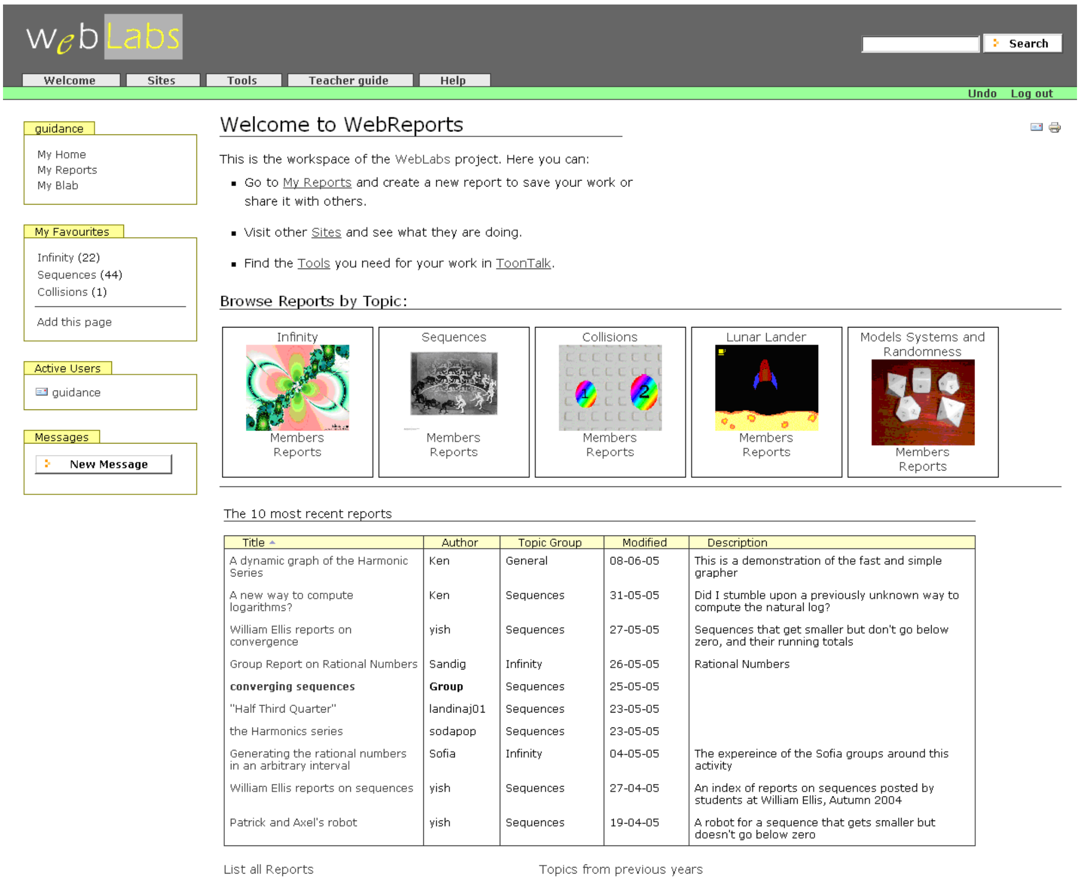


Figure 3: The ToonTalk Environment, showing the programmer's "hand" (1), a robot to be trained (2), and various tools used in training.

The user directly manipulates objects using a virtual 'hand' (labelled 1 in Figure 3), or with tools such as the magic wand for copying (labelled 8), vacuum cleaner (9) for cutting, pasting and erasing or bicycle pump (10) for changing object size. Programs are created by training a robot – directly leading it through the steps of a task it is required to perform. The robot remembers what it is trained to do, but only for the specific set of values with which it was 'trained'. These are stored in the robot's thought bubble (7). The robot's memory can then be generalised by 'vacuuming'; that is, by erasing the values and leaving an empty slot for 'any value'. Thus the concept of variables is introduced implicitly through the programming metaphors. Distinct program units (or modules) communicate using the bird-nest mechanism: a robot which completes a computation can pass the result to a bird, which would carry it to another robot for further processing. Needless to say, this mode of programming is very different from that used in traditional text-based languages, and induces different patterns and styles of problem solving.

5.2.2.2 WebReports

WebReports (Figure 4) was a web-based collaborative authoring system designed by myself and developed with the help of members of the WebLabs project. This system embodied many of the features now commonly referred to as "web2.0 principles" long before those became popular; it supported publication of user generated content and social dynamics around that content. The details of the system's functionality and its evolution are the focus of sections **Error! Reference source not found.** and **Error! Reference source not found.**.



Welcome to WebReports

This is the workspace of the WebLabs project. Here you can:

- Go to My Reports and create a new report to save your work or share it with others.
- Visit other Sites and see what they are doing.
- Find the Tools you need for your work in ToonTalk.

Browse Reports by Topic:

The 10 most recent reports

Title	Author	Topic Group	Modified	Description
A dynamic graph of the Harmonic Series	Ken	General	08-06-05	This is a demonstration of the fast and simple grapher
A new way to compute logarithms?	Ken	Sequences	31-05-05	Did I stumble upon a previously unknown way to compute the natural log?
William Ellis reports on convergence	yish	Sequences	27-05-05	Sequences that get smaller but don't go below zero, and their running totals
Group Report on Rational Numbers	Sandig	Infinity	26-05-05	Rational Numbers
converging sequences	**Group**	Sequences	25-05-05	
"Half Third Quarter"	landinaj01	Sequences	23-05-05	
the Harmonics series	sodapop	Sequences	23-05-05	
Generating the rational numbers in an arbitrary interval	Sofia	Infinity	04-05-05	The expereince of the Sofia groups around this activity
William Ellis reports on sequences	yish	Sequences	27-04-05	An index of reports on sequences posted by students at William Ellis, Autumn 2004
Patrick and Axel's robot	yish	Sequences	19-04-05	A robot for a sequence that gets smaller but doesn't go below zero

List all Reports

Topics from previous years

Figure 4: WebReports front page. This is the view a user sees after logging in. It includes a personal navigation bar on the left, links to main content areas and a listing of recently published content.

The atomic element of the WebReports system was the webreport:[2] an online multi-media document created by a member of the community – researcher, teacher or student. The basic requirements for webreports were set in the initial bid of the WebLabs project (WebLabs, 2001). This states that webreports should be:

- Collaboratively constructed, dynamic, web-based multimedia reports of evolving understandings of a knowledge domain.
- Include working models, along with multi-media descriptions, interpretations and reflections, textual and graphical illustrative explanations, and reflective notes and guidance hints.
- Provide a link that, when followed, will reconstruct the original object in ToonTalk for further editing and inspection.

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[2] I use WebReports to refer to the system, and webreports to refer to the actual documents.

While the details of design and implementation were only realised over the course of the project's first year, this set of requirements already identifies a rich source of data, which can potentially provide insights into the process and outcomes of participant's learning trajectories.

5.2.3 Iterative design

The experimental phase of my study followed the four iteration structure (Pratt, 1998) described in Chapter Four, with the necessary modifications implied by the characteristics of my research setting.

5.2.3.1 Iteration 0, Autumn 2002: Bootstrapping

The aims of the bootstrapping phase were to gain some basic insights regarding the affordances and suitable uses of the chosen technology, and use them as a basis for designing an initial set of tools and activities.

In terms of ToonTalk programming, this implied an intensive period of experimentation with ToonTalk development. On the surface, this effort was dedicated to the development of tools for presenting and manipulating number sequences, which children would use in the course of their activities. I myself was in fact undergoing a constructionist learning process, adapting my previous programming knowledge to the ToonTalk environment. This process was guided by the close mentoring of my supervisor, Prof. Richard Noss, and ToonTalk's creator, Dr. Ken Kahn. Tools emerged from an initial sketchy scenario of educational activity, to which more details could be added as the tool design matured. In turn, the elaboration of the scenario into a concrete plan placed new demands on the tools, and often called for altogether new tools. Many of the tools developed in this process primarily served my own learning as well as group discussions among WebLabs researchers, and were consequently radically altered before they reached the classroom.

Alongside the ToonTalk tools and activity design, I explored the design of the collaborative medium to support these activities and the WebLabs project in general. Using an Extreme Programming approach, (Beck, 1999), I began by constructing a mock-up and using it to discuss various usage stories with fellow researchers. Following this, a first prototype was developed by myself and Gordon Simpson. This prototype was the collaborative environment for the activities of the exploratory phase. It also served researchers as a platform for collaborative design of tools and activities.

5.2.3.2 Iteration 1, Spring 2003: Exploratory

The aim of the exploratory phase was to validate and elaborate the initial designs of tools and activities. The details of the experiment group are listed in Table 1 above. Evaluation in this phase was focused on the tools' usability and aptness, and less so on learning trajectories. The methods used were predominantly heuristic (Nielsen & Molich, 1990; Nielsen, 1994): several researchers observed children working through the tasks and took notes. These notes were used by me to identify design flaws. My conjectures regarding these flaws and possible remedies were discussed with colleagues in the project team and eventually a new design of tools and activities emerged. I was assisted in observations by Gordon Simpson, Constantia Xenofondos in London and Eugenia Sendova, Liliana Moneva and George Gachev in Bulgaria. Gordon Simpson, Ken Kahn, Richard Noss, Celia Hoyles and Eugenia Sendova contributed to the design discussions.

Evaluation takes *activity* as a unit of analysis (Kaptelinin and Nardi, 2006). Thus the resulting conjectures could lead to redesign of tasks, mediating tools, or context. For example, observing that task A is harder than expected and B easier, could suggest that the order of these two needs to be changed. On the other hand, seeing that constructing a particular tool is too taxing for learners could lead to the provision of that tool upfront. Design and evaluation were tightly interwoven in this phase. Often contradictions or tensions were identified in one session leading to the redesign of a task or a tool, which were tested the following week with another group or even with the same.

In order to sustain this responsiveness, the collaborative system used in this phase was based on a Wiki. This meant that the system could be rapidly reconfigured, albeit at the cost of cognitive overhead to the users. The fluidity of the system design meant that the data collected was extremely messy and hard to use as conclusive evidence for learning. Nevertheless, the insights gained from this iteration identified the main themes to be monitored in the next two.

5.2.3.3 Iteration 2, 2003 / 2004: Developmental

The developmental iteration took place in Autumn 2003 and Spring 2004. . The details of the experiment group are listed in Table 1 above. This iteration marks the shift of focus from technology and activity design to the trajectories of learning. At this stage, the design was stable enough to afford the collection of reliable data. Modest predictions were derived from the heuristic observations of the previous stages and compared with empirical results.

In the transition from the previous iteration to the present, the exploratory task fragments were woven into a coherent plan of activity spanning two school terms. This activity plan was driven by clearly defined learning aims, and milestone tasks directed at evaluating these aims. Design benefited from a clear plan on the one hand and grounded intuitions regarding tool adequacy on the other, resulting in tools with good fit-for-purpose. Consequently, the time spent on technical issues was minimized and more attention could be devoted to tracing the learning trajectories.

The collaborative platform was also replaced. The Wiki used as a malleable prototype served as a model, and along with the specific requirements derived from the activity plan, fed into a detailed specification of a highly structured platform (implemented by Jakob Tholander and Jesper Holmberg from The Royal Institute of Technology, Sweden - KTH). The key principle in its design was functional minimalism: include only the features and options which are needed to support the activities and practices of the community.

The tight link between pedagogic and technological design was maintained, but at a higher level. Frequent adjustments of design – pedagogic, technological and methodological – were generally avoided. Instead, systematic refinement was driven by the evaluation of the plan of activities as a whole.

5.2.3.4 Iteration 3, 2004 / 2005: Analytical

The analytical phase relied on stable and mature activities and tools. While occasional local refinements were not ruled out, the main design challenge was to identify and adapt methods of data collection and analysis, suitable for the context of study and the technology developed to support the activities. The specific instruments are the focus of section 5.3.3.

5.2.3.5 2005 / 2006: Retrospective analysis

With the completion of the empirical cycles of design and evaluation, the time had come for retrospective analysis from a broad perspective. This phase was thematically driven, tracing several questions through the data collected across all iterations. The aim of this analysis was both to substantiate claims regarding the learning process and to elucidate elements of transferable design knowledge.

In terms of the epistemic themes, these were mapped to existing theory of mathematical learning and the evidence was interpreted in that context. Consequently, the instruments used were geared towards the evaluation of mathematical performance and mathematical discourse.

Some of the prominent design elements were captured using a framework of design patterns (Alexander, 1979). This work was partially supported by the Learning Patterns project (Kaleidoscope JEIRP), directed by David Pratt and Niall Winters.

5.3 Collection and Management of Data

Chapter Four characterised design research methodologies as iterative, process-focused, interventionist, collaborative, multileveled, utility oriented, and theory driven. As a corollary the appropriate methods of data collection highlighted process-oriented observations, using a mixture of mainly qualitative methods, including video and audio recording of student activities, analysis of texts and artefacts produced in the course of these activities, interviews and ethnographic field notes. This versatile process oriented approach to data collection dominated my research.

5.3.1 Sources of Data

My study draws on three classes of data: design data, student productions, and classroom observations. Design data refers to the details of the tools and activities which I have designed. It encapsulates not only the final form or design, but also the path which led to it. Student productions are the actual artefacts, or multi-modal texts, produced by students in the course of activities. These include written text, ToonTalk models, spreadsheets, charts and graphics. Classroom observations include audio, video and field notes recorded by me and my colleagues during activities. The particular experimental setting I worked in had its opportunities and challenges, which are reflected in the primary sources at my disposal.

5.3.1.1 Design Data

Design data were collated from WebLabs on-line guidance and project reports which I authored. Data reflecting the design process was retrieved from on-line discussions with my colleagues, various design drafts and sketches, and from my research journal.

5.3.1.2 Student Productions

Student productions were predominantly collected from webreports produced by students. The WebReports system was designed to support students' investigation, with the important side effect of capturing their reflections, in their own words, at key points of the learning process. I make extensive use of this source. Apart from the text, students' reports included graphics and programs.

All of these were treated as means of expression that provide a window on students' evolving concepts. The nature of ToonTalk allowed me to see through the code and interpret the explorative process by which it was conceived. In several cases, paper-based tasks or questionnaires were also collected.

5.3.1.3 Observations

By and large, observations were done during active facilitation of activities, and are thus highly participatory. Video and audio collection was often restricted by the experimental settings. Consequently, *in-activity probes* play a central role in observational data: short (up to 5 minutes) unstructured interviews taken while students were engaged in an activity. These probes aimed at capturing snapshots of the process of knowledge construction. In order to enhance the reliability of these data, they were calibrated with field notes and with the observations of my colleagues.

5.3.2 Data Cataloguing

Design data were filed chronologically and thematically. Student productions in the form of webreports were automatically catalogued in a searchable database. Paper based productions were filed chronologically and thematically. Observational data were indexed, annotated and partially coded. Audio and video recordings were selectively transcribed. Transcriptions emphasised the content of expressions, with less attention given to gesture and emotive facets of discourse.

Data were analysed in two modes: situated and reflexive. Situated analysis refers to the attempt to interpret data as it unfolds and respond to emerging issues. At the micro scale, this could mean on-the-spot design adjustments. More often, this would lead to adjustments and resampling, from one session to the next or between design iterations. The reflexive mode concerns in depth analysis of data after the completion of an experiment, with the added perspective of time and the opportunity to compare data across longer spans of activity. Situated analysis focused on identifying indicators of learning and conflict, proposing preliminary explanations, and verifying these by subsequent interventions. The themes which emerged from this mode were theorised and used as a frame of reference for the reflexive mode. Reflexive analysis tracked these themes across incidents and data-forms.

5.3.3 Data collection

Section 5.3 opened with some principles of data collection and analysis in design based research, and some challenges they implied. These challenges are amplified by field conditions: a nearby building site makes audio recordings worthless, bad lighting conditions eliminate the possibility of video recording. A planned pre-test is cancelled by unexpected change of school schedule. In the midst of these is the complex position of the participant-observer, trying to interact with learners and record these interactions at one and the same time.

Several principles emerged in my attempt to meet these challenges: redundancy, triangulation, and nearest substitute.

Redundancy means that anything available is collected from the scene. Recordings, notes, produced artefacts, scraps of paper scribbled on by learners or researchers: everything is saved, even though little may be used. Many of these items may have no use as data, but they can still contribute as memory aids when constructing post-hoc descriptions.

The inevitable sparsity and discontinuity of data is a challenge to validity. Triangulation (Bell, 1998; Denscombe, 2003) tries to meet this challenge by juxtapositioning evidence obtained by different methods. For example, the interpretation of an interview transcript can be supplemented by field notes describing the context in which it was taken and by analysis of the artefacts constructed by the learner prior to the interview.

The term 'nearest substitute' acknowledges the pragmatics of the research setting, and accepts the use of instruments which are as close as possible to the ideal. When video recording is ineffective, it is substituted by audio. When even that is infeasible, verbatim notes of key oral expressions are taken as soon as possible after the event.

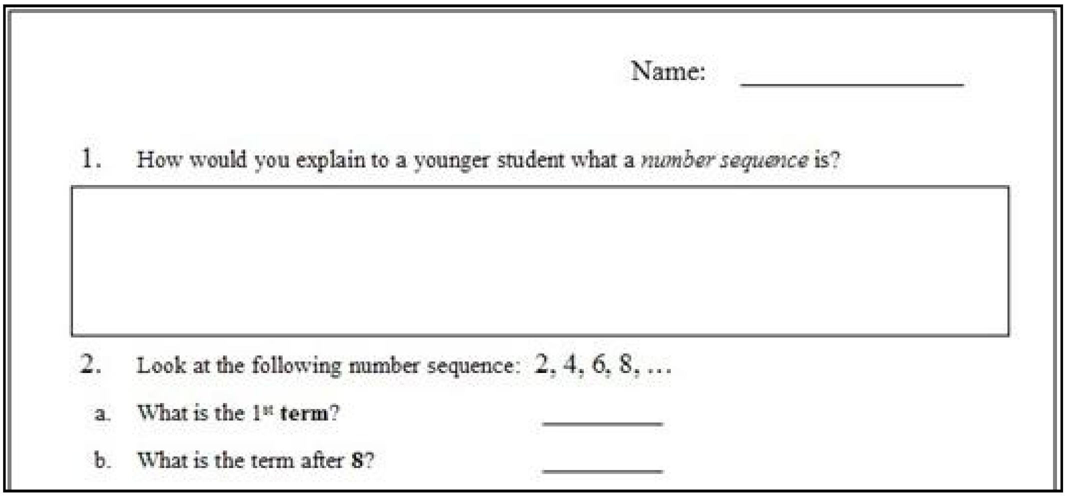
The actual instruments of data collection used in this study included:

- Pre- and post-trial written evaluations and interviews.
- Notes and recordings (video and audio) of learners' face-to-face discussions and classroom presentations.
- Stimulated recall interviews (Lyle, 2003), in which students were provoked to share their reflections on the activities and their products, and express the conceptualizations they have developed through them.
- Task-based interviews (Koichu and Harel, 2007) in which a learner is presented with a task and prompted to discuss it as she performs it. This method is designed to test specific conjectures of thinking-in-change in near-laboratory settings.
- In-activity probes (Mor et al, 2005), short interviews – typically up to five minutes – conducted while a student is engaged in an activity and referring to it. The use of this tool aims at capturing the process of knowledge construction and allows students to express their situated abstractions in the context that they are formed.
- The multimodal (Jewitt, 2003) text of learners' webreports and task worksheets, including their comments on peer reports.
- The ToonTalk code produced by learners, as published in webreports or collected from their workspace.
- Field notes recording students' work process as they perform tasks or participate in discussions.

Section 5.3.3.1 offers examples of the various data types, while sections 5.3.3.2, 5.3.3.3 and 5.3.3.4 highlight some specific issues which emerged in the process of collecting these data.

5.3.3.1 Examples of data sources and methods of analysis

The examples shown here are drawn from the first phase of group IV's basic number sequences activities (XX). Students were administered a pre-trial questionnaire, and then proceeded to work on the add-a-number task. While students were working on the task, they were invited one by one for a stimulated recall interview, in which their answers on the questionnaire were reviewed. As they were working on the task itself, Gordon Simpson and I conducted in-activity probes. After each session we produced a session report based on our field notes. After completing the task, the students published a webreport which included their models, observations and answers to a few questions.



Name: _______________

1. How would you explain to a younger student what a *number sequence* is?

2. Look at the following number sequence: 2, 4, 6, 8, …

a. What is the 1st term? ________

b. What is the term after 8? ________

Figure 5: Fragment of Group IV's basic sequences pre-trail questionnaire (3 & 9 Nov. 2004)

Figure 5 shows the top of the first page of the pre-trial questionnaire as it was printed and presented to students. Students completed this questionnaire in class, and the completed forms were collected and processed by the next session.

	Michael		Sam		Harold		Luthar	
1	A number sequence is about 3 or more numbers, in a line that have something in common E.g. 1, 2, 3, 4 you keep on adding one to the numbers	O	some numbers that follow a certain pattern	P	A number sequence is like a row of numbers which follow each over(sic), like 1, 2, 3, 4 or 2, 4, 6, 8. Because they all go up by the same amount	A	It's a series of numbers that have rule that will change them	R
2		C		C		C		C
a	2		2		2		2	
b	10		unclear - either 10, or 16		10		10	

Figure 6: Coded responses to the pretest (9 Nov. 2004)

Figure 6 shows a section from the coded responses to the questionnaire in Figure 5. The relevant code significations, with examples from the whole group, are provided in Figure 7.

1: How would you explain to a younger student what a *number sequence* is?			
1		P	A sequence is a pattern
			A pattern of numbers progressing by the same amount each time
			A number sequence has a pattern
1		A	A sequence is an arithmetic progression.
			a number sequence is when you keep adding a number to another one
1		R	A sequence is defined by a rule
			It's a series of numbers that have rule that will change them
1		N	No significant definition
			A number line
1		O	Other notable statement
			A number sequence is about 3 or more numbers, in a line that have something in common E.g. 1, 2, 3, 4 you keep on adding one to the numbers
2. Look at the following number sequence: 2, 4, 6, 8, … a. What is the 1st term? b. What is the term after 8? c. What is the 10th term? d. What is the 100th term?			
2	a-d	C	Correct, know what a "term" is
2	a-d	W	Wrong, do not know what "term" means

Figure 7: coding table with examples

Figure 8 lists a raw transcription of the first few minutes from a stimulated recall interview conducted with Luthar during the following session (note that both Luthar and I had mistaken the date).

	Y	I've started recording you, now, ok, so, just Luthar, right, and what date is it today, 13th?
	L	Think it's the 12th, I'm not sure.

	Y	12th, OK, 12th of November. What we're going to do, is this. We're going to go through this questionnaire which you filled last week, OK, and I'm going to ask you to clarify a few of your answers here, and then we're going to, I'm going to give you another task to work on, and we're going to talk a bit about that, OK?
	L	OK, fine
	Y	Um, right, so, number sequence is a series of numbers that have a rule that will change them. OK, can you explain what you mean by that?
	L	Um, it's like, uh, well, if I use an example like if you have, if it's the first one in the sequence, and then it's the number 3, it could be n plus 3, could make the sequence, so it's, um, uh, it's a, like a chain of numbers that, with each one changing to a rule, so not just randomly.
	Y	OK. I just, I [unclear] maybe. Sorry, OK, sorry, back to this, OK. So here, how did you work out the 100th term, so you write 20 times 10. Why did, why does 20 times 10 help you work out the 100th term of this sequence?
	L	Um, [whispers] true [whispers] not sure, cos, for instance, if you take 1, then if you times that by 2, you get 2 so if you want to work out the 100th term, you times, so 1 by 100 makes it 100, so you times, um,
	Y	Can you? How would you name this sequence, how would you call, what would you call it?
	L	2n
	Y	2n. Right, so does that, when you call it 2n does that help you explain what the 100th term is?
	L	Yeah, because if n is 100, then you times it by 2. That would have been easier, but.

Figure 8: stimulated recall interview with Luthar (16 Nov. 2004)

Figure 9 presents the text of the session report produced by me after the session during which the interview in Figure 8 was conducted.

Group IV.

Visit 16th November, 2004

1 hour (afterschool) session 3:30-4:30 pm

Six 13-14 yr-old boys.

One computer per student.

Teacher: David Croston

Students: Sam, Paul, Jon, Luthar, Alan (new), Aaron (missed last week)

Students created accounts on WebReports by following through a demonstration on the projector (minimum of hassle, these kids are more net savvy than our previous groups). We gave a very brief introduction of the system. Then we showed them the add-a-number webreport template and explained they had to download the ToonTalk object and complete the task. Note that this was their first session in ‘freeplay’. We gave a demonstration in ToonTalk of what the task consisted of, introduced bird-nests (they hadn’t seen them before) etc then set them going on the task. Yishay conducted the pretest interviews with Sebby, Luke and Adam (now transcribed by Sophie). Gordon helped students with the task and conducted some brief in-activity probes (audio files filed and labelled).

There were four major issues that came up during the programming:

Using the wand. In order to complete the task, students need to **copy** the *add this* onto *current*, and then **copy** *current* and give it to the bird. In most first attempts, students would **move** one or both of these number pads instead of copying them, resulting in the numbers not being available to the robot the next time he tries to run. With a bit of help, students came up with the idea of using the wand.

Training robots once or more than once. Some students still aren’t sure about whether they need to train a robot’s actions once or more than once (this is a common problem when students are first learning). One student trained their robot to go through two iterations of copying *add this* and *current* etc, “just to make sure he got the idea”. This is related to the next point – robots often stop after one iteration because they need to be generalised, and students sometimes infer from this that robots will only run once if they were trained once.

Generalising. Students have to generalise their robot by erasing (or sucking) the *current* after training. Most got this idea, although some were initially confused as to why the robot stopped after one iteration. Had a good discussion in the group at the end about this topic (i.e. exactly what an erased number pad means in a robot’s thought bubble).

Leaving things in the input box. Perhaps a less important issue, but some students trained their robot to take the bird out of the input box and put it on the floor, before giving it the *current* number. After the robot runs the first time, he will get dusty and vacuum up the bird (because robots are tidy). One student asked if you can train a robot to not be tidy, and I had to tell him this wasn’t possible!

Towards the end of the session we asked students as a group how to program the robot. Patrick did most of the instructing, as he was the only one to have successfully programmed his robot (with Gordon’s help). Note that David answered quite a few of the questions we were directing at students!

Note: got a few permission slips back for the 6th, David is holding on to them.

Technical issues:

Dinesh opened up the ability for students to download (from webreports, but actually it was from any site I think) which was good. Unfortunately he has to turn this on before every session, and turn it off afterwards. Similarly he hides the ToonTalk icon between sessions and we have to rely on him being there and setting things up for us before each session which is not ideal.

The upload tool is not working. Could be either because FTP is blocked, or because the ftp exe hasn’t been given the privileges to run. Should be able to work around by uploading the locally saved tt file using the old browse mechanism on the webreports site (which uses HTTP).

Figure 9: session report (16 Nov. 2004)

Figure 10 shows Luthar’s first webreport, published after he completed the add-a-number activity.

Add a number

Created by **luminardi** - Topic Group: **Sequences** - Created: 16-11-04 - Modified: 06-12-04

Training Add number

I trained a robot to repeatedly add 1 to produce the numbers 0, 1, 2, 3.... Call the robot *Add number.*

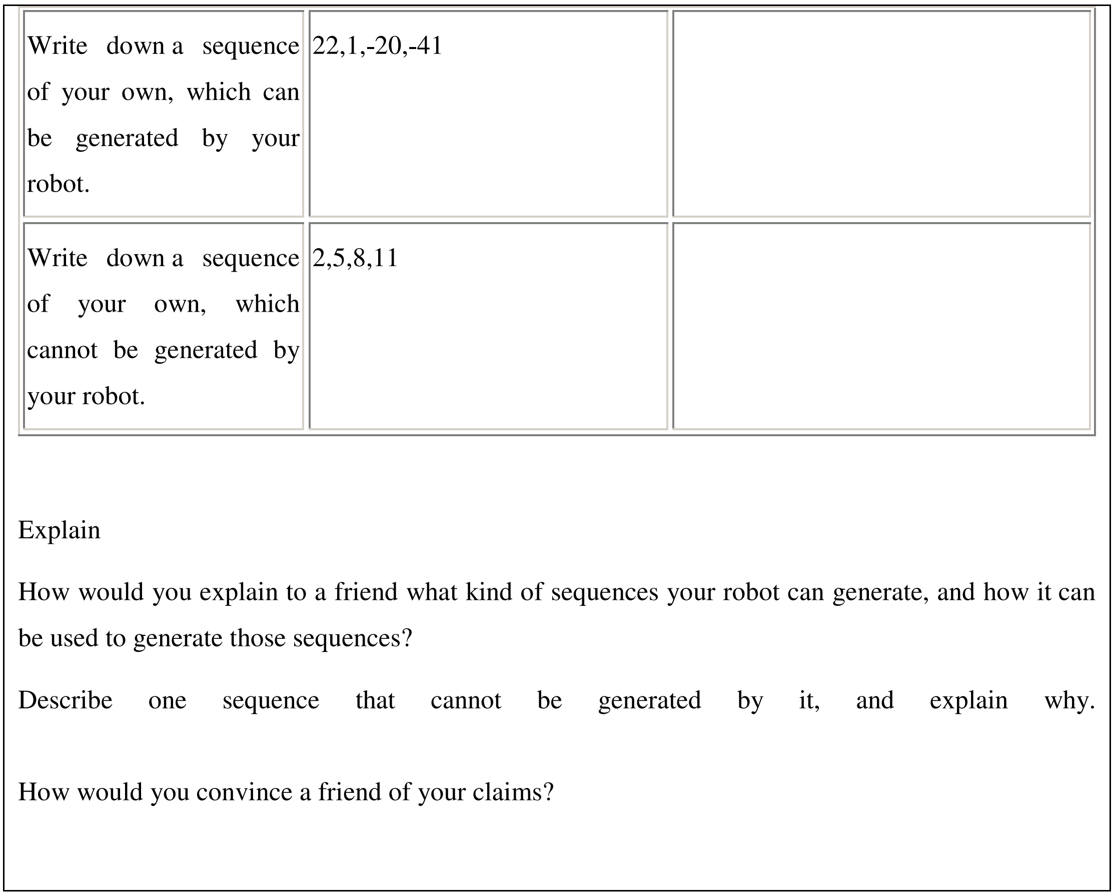
Explain how you trained the robot:

What I did to make the number's go up by one each time is I made the robot add the 'add this' to the current then I copy it and give it to the bird

Explore

Can you think of a way to use your robot to produce these sequences?

Sequence	Explain - in words	Explain - in ToonTalk
	If yes, explain how you would do it. If you think it is impossible, explain why.	Add any ToonTalk object that helps show how you did it. Describe what a robot must do to produce a sequence.
2, 3, 4, 5…	yes just make the current 1	
-1, -2, -3, -4…	yes just make the add this -1	
-7, -6, -5, -4..	yes just make the current -7	
2, 4, 6, 8…	yes just make the add this 2	
5, -1, -7…	yes just make the current 5 and add this -6	



Write down a sequence of your own, which can be generated by your robot.	22,1,-20,-41	
Write down a sequence of your own, which cannot be generated by your robot.	2,5,8,11	

Explain

How would you explain to a friend what kind of sequences your robot can generate, and how it can be used to generate those sequences?

Describe one sequence that cannot be generated by it, and explain why.

How would you convince a friend of your claims?

Figure 10: Luthar's first webreport

Finally, Figure 11 offers the raw transcription of an in-activity probe conducted with Luthar as he was working on the next task (`Add-up`, section XX). The total length of this probe was a minute and a half.

	Y	So, you already explained why in the in box, it actually tells you how many numbers you've added, ok.
	L	Yeah
	Y	And then the total, what does the total show you?
	L	And that shows, what the, in, all the numbers, up to this one have made the [unclear] so the in box.
	Y	OK, what, can you say anything about the relationship between the number you have in total, and the number you have in in?
	L	Probably something to do with 3, because we've got a sequence of triangle numbers, so, probably this is something the, relate, is this times that 3? Uh, no.
	Y	What is this and what is that? That…
	L	Total is, this is total, and that was in, so that…
	Y	Yes
	L	So total, the in times 3 doesn't equal the total, I thought it might.

	Y	It's not in times 3, you say,
	L	No
	Y	So what would it be?
	L	Not sure. I could do total divided by in to find out. Not sure how you do that in ToonTalk.
	Y	Do you think there'll be a constant?
	L	It'll probably be changing every time, because the number that it's adding by changes every time, so.
	Y	Alright

Figure 11: In-activity probe with Luthar (6 Dec, 2004)

Luthar has just finished constructing a robot which takes the natural numbers sequence as an input, and produces a sequence of their partial sums. The purpose of this probe is to establish what Luthar had noticed, and understood, regarding the mathematical features of the mathematical process embodied in the robot and the resulting structure. The questions provoke him to suggest various conjectures (e.g., the output is the "times 3 table") and verify or refute them. Inter alia, he makes several mathematical arguments. The most significant is the last: the resulting sequence cannot be linear because "the number that it's adding by changes every time".

5.3.3.2 Observational instruments: video and audio recordings and field notes

Due to the sound and light conditions at schools, video recording on site was typically inefficient. Video was used predominantly during five full-day workshops, collecting four to six hours of footage in each. This footage was scanned and indexed to identify key incidents which were transcribed and analysed in greater detail. Recordings were assisted by Richard Noss, Gordon Simpson and Constantia Xenofondos.

Audio recordings were used rarely in group I, regularly in groups II, III and V, and extensively in group IV. The three common formats were group discussions, work sessions and interviews, discussed below. Table 2 provides a summary of recordings by group, data and format. Group discussions ranged from five to thirty minutes. Longer discussions were recorded at the inauguration and conclusion of every segment of activity. Shorter discussions were often opportunistic: exploring an issue which emerged from the students' work. Discussions held at the lab (during workshops) were recorded on video. Altogether, between four and ten discussions were recorded per group.

Work sessions were incidents where one of my colleagues or myself worked with an individual or a small group of students on a particular programming task. Such sessions would typically run for five to ten minutes, during which a recording device was placed unobtrusively before students.

Video and audio recordings were indexed, partially transcribed and partially coded.

group	topic	session	recordings	format
II	basic sequences	22 Jan 2004	1 (23 min)	group work session
II	Plotting sequences	12 Feb 2004	1 (20 min)	group work session
III	convergence	13 Feb 2004	1 (30 min)	group discussion
III	convergence	04 March 2004	1 (10 min)	group discussion
III	convergence	23 April 2004	1 (33 min)	group discussion
III	convergence	30 April 2004	1 (32 min)	group discussion
IV	sequences	9 Nov 2004	4	pretask interviews
IV	sequences	16 Nov 2004	9	pretask interviews and IAPs[3]
IV	sequences	23 Nov 2004	5	IAPs
IV	sequences	30 Nov 2004	7	work sessions, discussions
IV	sequences	6 Dec 2004	13	(workshop) discussions, work sessions,
IV	sequences	14 Dec 2004	10	IAPs
IV	convergence	22 march 2005	8	intro discussion, IAPs
IV	convergence	12 April 2005	7	discussions
IV	convergence	19 April 2005	10	IAPs
IV	convergence	17 May 2005	8	IAPs
IV	convergence	23 May 2005	7	(workshop) discussions, IAPs
IV	convergence	24 May 2005	10	conclusive interviews, IAPs, discussions
V	convergence	5 June 2005	30	IAPs, discussions

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[3] IAP = in activity probe. See 5.3.3.3.

Table 2: summary of audio recordings

Shorthand notes were taken during sessions, with verbatim quotes from learners' oral expressions, along observations of their activity and affective state. These notes were elaborated into detailed reports shortly after the session, and supplemented by relevant excerpts from their produced texts and code and transcripts of audio recordings when available. These reports were then reviewed by fellow researchers who had been on site, and any discrepancies in observations were discussed.

5.3.3.3 Interviews

Several forms of interview were used, predominantly in conjunction with other instruments.

Stimulated recall interviews (Lyle, 2003) provided a form of post-hoc observation, enhancing the understanding derived from products or passive observations, by eliciting learners' perspective on their actions. They were used mainly as a follow-up to written assessments (Phillips et al, 2000). At key points in the activity sequence, learners were presented with worksheets or questionnaires. After these were reviewed, some or all of the students would be interviewed and asked to elaborate their responses and explain the thought process behind them.

Task based interviews were used mainly in the early iterations, to test specific conjectures regarding cognitive and epistemic facets of tool design. A single student would be asked to perform a task and explain her actions. This technique is similar to the 'think-aloud' method used in usability studies (Phillips et al, 2000; Barendregt et al, 2003).

In-activity probes (Mor et al, 2005) denote a technique which emerged from the WebLabs project. I would approach a learner in the course of a scheduled activity and conduct a short interview on her actions at the moment. This technique proved to be a highly powerful and cost-effective instrument, allowing me to substantiate observations and induce learners to articulate their thinking-in-change. To an extent, it used a standard instructor practice in constructionist classrooms as a form of a task-based interview. This allowed data collection to be streamlined into the flow of activity. Yet it calls for special care: observation needs to be non-suggestive. If students mistake an act of observation for intervention, they might misinterpret it as affirming their conjectures. The distinction between observation and intervention is easy to maintain, by stating explicitly ("I'm not saying if this is right or wrong, I just want to hear what you think"), or by gesturing at a recording device. Often the probe was followed by an intervention, for example, after students were interviewed, they would occasionally ask for feedback on their responses, which would lead to a didactic discussion of their work.

5.3.3.4 Texts and artefacts

Texts and artefacts produced by learners, in various media and modalities, were used both for formative and summative evaluation.

Paper and pencil questionnaires were used in early iterations for summative evaluation. A worksheet with questions reflecting the subject domain was presented to learners before a section of activities, and a similar one used at its end. With time, this instrument was modulated in two significant ways.

The first change was driven by the realization, discussed in **Error! Reference source not found.**, that any act of evaluation is inevitably an intervention. Consequently, these questionnaires were integrated into the design of activities, as a means of introducing a subject or as a prelude to a concluding group discussion.

The second change was a typical example of the dynamics of iterative design. As the WebReports platform matured, new possibilities of using it emerged. One of these was the Active Worksheet, a webreport template which serves the two-fold process of directing students' attention towards the ideas and explorations in which we are interested as well as providing valuable data through the students' responses to the questions posed. Using a webreport for this purpose was a practical convenience, but it also allowed learners to embed various digital artefacts in their text, such as graphs, sketches and ToonTalk objects.

Webreports were originally conceived as a deliberate form of intergroup communication, strongly facilitated by teachers and researchers. Eventually they became a medium for both personal and collaborative expression, reflection and action (XX). As such, they were an indispensable window (in the spirit of Noss and Hoyles, 1996) on the process and outcomes of learning. Thus, with time webreports became a prime source of data. Table 3 shows a summary of webreports produced by students per group and activity. Group I was not counted because the reports they produced were too rough, and were used mainly to infer user interface requirements for the WebReports system. Group III was only introduced to WebReports in the course of the convergence activities. Group reports were collaboratively authored by students, assisted by me or my colleague Gordon Simpson, at the end of an activity segment.

Group	Number of Students	Number of webreports		
		basic sequences	Convergence	group reports
II	6	15	4	2
III	8		12	3
IV	10	26	15	5
V	5		4	1

Table 3: webreports by group and activity

In addition to these, over 50 reports were collected from the *Guess my Robot* activity segment (XX). These were collected from eight schools, over two years.

5.3.4 Analysis

The first level of analysis is oriented towards the fundamental question of epistemic outcomes. Before asking how students learned, and how the design of activities and tools contributed to learning, we need to verify that they did actually learn something. The primary sources of evidence in this respect are the mathematical objects they produce and the mathematical arguments they articulate. Products were analyzed in terms of their aptness (Jewitt and Kress, 2003) for the task in hand, and their complexity and sophistication.

The term *Mathematical argument* is taken to signify any deliberate expression aimed at conveying mathematical ideas or claims to an audience. From a situated abstraction perspective, mathematical arguments are not restricted to the conventional formal language of mathematical science. More often they would be stated in a form or medium derived from the context of activity. Consequently the method of analysis assumes that learners engaged in an activity of mathematical nature will attempt to make mathematical arguments, and researchers should aim to identify and understand them. In my analysis I look for the mathematical meanings that are constructed and expressed using the tools provided within the context of activities. My guiding assumption is that text is articulated for a purpose, and should be interpreted in the context of that purpose. In other words, in order to understand what the author of a report *meant*, we need to observe *what* she published in the context of *why* she published it.

The next level of analysis strives to unpack the process of learning and its relation to the activities learners were engaged in and tools which they used. In order to do this, special attention is given to learners' articulation as they confront the tasks. These are tracked in their reflective notes in webreports, in discussions between them, and by in-activity probes.

Both levels of analysis are present in two grain sizes: as an instrument for immediate, week-to-week refinement, and as part of the transition from one iteration to the next. The third level is that of reflective analysis, subsequent to all iterations. This level is focused on identifying and elaborating cross-cutting themes, and takes the outcomes from the first two as its raw data.

Chapter Four highlights the epistemic role of narrative, as a mediator between experience and paradigmatic knowledge. This observation prompts the use of design narratives as a scientific tool. It also serves as an analytic guideline: learners' expressions are interpreted as narratives, whether they are presented in words, image or code.

5.4 From Data to Design Narratives

The data harvested throughout the cycles of design experiments are eclectic, opportunistic, and sporadic: the challenging and unpredictable research environment requires that any possible form of data is used, resulting in inconsistent quality and amounts of usable data. Often the most interesting events are unplanned, thus calling for "ad-hoc" responsive data collection. The transition to the retrospective phase calls for systematic organisation of these data. Design narratives provide a means to this end.

Bruner (1990; 1991) identifies narrative as the predominant tool by which humans organise events to derive meaning. The instrument of design narratives, as described in Chapter Four, aims to formalise this innate process into a more methodical one. In order to provide the transparency expected of a scientific method, several questions need to be considered:

- How is the set of design narratives pertaining to a study selected?
- How are events to be included in these narratives chosen?
- How are the factual claims contained in the narratives verified?

The remainder of this section addresses these questions.

5.4.1 Selecting and Constructing Design Narratives

The design narratives listed in Chapter Seven were selected by a two-phase process: first, a large set of candidate narratives was compiled so as to ensure chronological and thematic coverage. This set was culled so that each narrative would capture either a problem or a solution which was unique. Where duplicates were identified, priority was given to the candidate that was more representative and better supported by data. Consequently, this process addressed two of Bruner's principles, as discussed in Chapter Four: the individual narratives' *canonicity and breach*, and the *accrual* of the collection as a whole.

In the first phase, data was catalogued by year and activity, and then scanned to identify incidents which illuminate the central themes defined in Chapter Three: designing for learning about number sequences by *Construction, Communication and Collaboration*. A candidate for this initial set typically emerged from a single document: a design specification, project report, video or audio recording, web report, etc. During the selection process, additional sources were listed for each narrative. Once a narrative was chosen to be included in the final set, these sources were used to calibrate the data from the initial source and fill in any gaps. A second selection criterion was the need to balance researcher narratives and learner narratives. Researcher narratives refer to the process of designing tools and activities for learning, whereas learner narratives relate to learners experiences with the tools and the activities. This distinction is elaborated in Chapter Seven. A template was used, to ensure that all the necessary elements are present in each narrative. This template is described in detail in Chapter Seven. The next section explains the rationale behind it.

5.4.2 Structure and Form of Design Narratives

Chapter Four considered Bruner's ten qualities of narrative, and their mapping to design narratives as a form of scientific discourse. Seven of these principles were translated into concrete guidelines used to develop the design narratives, and manifested in a common template used to structure these:

- *Diachronicity*: each narrative recounts a single, contained, thread of events.
- *Particularity*: preference was given to detailed description of representative incidents over cumulative generalisations.
- *Intentional state entailment*: the apparent intentions driving the narratives are declared explicitly. These are either the educational aims behind a particular activity, or the learner's task derived from such an activity. As noted in Chapter Four, intentional states are inferred, not observed. My own inferences are presented as an epilogue to each narrative, so as to open them to criticism.
- *Hermeneutic composability*: similar to intentional states, the body of the narratives were left free of commentaries, but my reflections were included in the epilogues. These reflections weave the narratives into a larger story.
- *Referentiality*: in contrast to fictional narratives, the design narratives in this study need to refer convincingly to real events. Furthermore, they provide an "audit trail" (Creswell and Miller, 2000; Lincoln and Guba 1985) by listing the sources used in their construction.

- *Normativeness*: The normative claims derived from the narratives are expressed in Chapter Eight in the form of design patterns.
- *Context*: the common context of all narratives in this study is provided section 5.2, and elaborated for each narrative in its preface.

These principles are manifested in the design narrative template. The core of this template is a *STAR* structure: *Situation, Task, Actions, Results*. The *Situation* element conveys the context, the *Task* element puts forth the intentional state, Diachronicity guides the *Actions* section, and the normativity of the actions is implied by the *Results*. The STAR structure is augmented by a *Sources* section which addresses referentiality, and by a *Reflections* section, which makes explicit hermeneutic inferences, intentional state entitlements and normative claims.

5.5 From Design Narratives to Design Patterns

A common perception in many professional communities sees design patterns as elements of craft lore, expert intuitions made available to a practitioner community. In order to include design patterns as elements of a scientific discourse, a clear path needs to be visible from narratives to patterns, and mechanisms established for validating them. In the case of this study, the process used in this study included the following steps:

1. A prominent design feature was identified in a design narrative, and linked to a desirable outcome, or to the resolution of a critical problem.
2. The design feature was captured using a core template of *Problem, Context,* and *Solution*. The source design narrative was noted.
3. Other narratives were searched for additional support.
4. The problem was expressed as a configuration of forces (as explained in Chapter Eight).
5. The initial context of the pattern was defined by the situational characteristics common to all supporting narratives.
6. The solution was articulated in the most specific detail that still encompasses all supporting cases.

The same collection of design narratives could, theoretically, have given rise to many different sets of design patterns. Again, the primary yardstick was the question derived from Aim 3 in Chapter Three: how to design for learning about number sequences by *Construction, Communication and Collaboration*. Thus, the initial set of patterns expressed insights addressing this question directly as they emerged from the design narratives.

The identification and articulation of the initial set of patterns was followed by a phase of organising and refactoring the pattern language as a whole. The links between patterns were identified and noted, and new patterns were derived by structural manipulations, such as:

- *Generalisation*: when several patterns were recognised as variants of the same idea, these common elements were expressed as a pattern at a higher level of abstraction and the two noted as its extensions.
- *Specification*: when a pattern's empirical support was recognised as insufficient, the pattern's scope was narrowed down to fit the evidence.

- *Decomposition*: patterns which were too complex or too sensitive to contextual factors were broken into several more robust components, each expressed as a separate pattern.
- *Extraction*: design features which recurred in several patterns were expressed as a new pattern and noted as a component in the others.

This process was iterated until it produced a stable collection of linked patterns. Patterns which lacked sufficient empirical support, or were poorly connected to the collection, were eliminated. The guiding objective was to collate a coherent set of patterns, offering a solid base for a potential language of patterns for technology-enhanced environments for learning mathematics through Construction, Communication and Collaboration.

The patterns which were produced by this process were then substantiated further by eliciting empirical and theoretical support from the literature. Finally, visual aids such as metaphoric illustrations and structural diagrams were added to enhance the patterns' text.

5.6 Summary and Conclusions

This chapter addressed Aim 2 by tracing the methodological framework and instruments of the demonstrator study, from data collection and management, through interpretation and systemisation of observations as design narratives and on to the formalisation of research outcomes as design patterns. This methodological framework was derived by projecting the principles and constructs proposed in Chapters Two and Four onto the research question and in the context of the research settings of the demonstrator study. Thus, this chapter bridges between the primary study of the thesis and the demonstrator study which validates it.

The chapter began with a description of the experimental setting: the classroom environment, the technological setup, and the process of iterative design. It proceeded to list the methods of collecting, cataloguing and analysing data used in this context. Finally, it articulated the process by which design narratives were constructed from the data, and design patterns extracted from the narratives.

Taken together, the result is a full specification for implementation of the analytical hemicycle of the design experiment cycle proposed in Chapter Four (Figure 2), and of the retrospective analysis phase of the design research meta-cycle (Figure 1). While the principles and constructs presented in previous chapters claim to be generic (to a degree), the instruments described in this chapter are a single instance of their application to a given problem domain and experimental circumstances.

Three classes of data were identified: design data, student productions, and classroom observations. Design data include any record of the design process and its product. Student productions refer to multi-modal texts and artefacts produced by students in the course of activities. Classroom observations denote any account or recording of students activities. The main focus was on process data, with occasional pre / post assessments where relevant. The challenges of a messy environment were addressed by:

- **Redundancy**: collect any bit of evidence offered by the scene of activity.
- **Triangulation**: juxtapositioning evidence obtained by different methods.

- **Nearest substitute**: accept the limitations of the research setting, use pragmatically available data which is closest in form to the ideal.

The primary sources for design are project reports, design documents, teacher manuals and research journals. The primary sources for student productions are student webreports, ToonTalk code and paper-based written tasks. All texts and artefacts were read as mathematical arguments expressed in narrative. Acknowledging the impossibility of separating observation from intervention, data collection was integrated with activity design – e.g. pre tests became motivators for new topics. Products were assessed in terms of aptness, complexity and sophistication of argument. The primary sources for classroom observations were field notes, video and audio recordings. Interview data included (individual and group) stimulated recall interviews, task-based interviews and in-activity probes. The latter played a central role in observational data.

A structured process of selection and construction of design narratives was identified, using Bruner's ten principles as guidelines. These principles, adapted to the needs of scientific form, were expressed in the design narrative template.

Design patterns were extracted from design narratives through a six step process devised to capture the key design elements, systemise and substantiate them. This was followed by a phase of refactoring: structural manipulations which give the pattern language as a whole greater coherence.