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Yishay Mor, phd@yishaymor.org

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Chapter 1 Introduction

This chapter begins with some thoughts that inspired and motivated my study. It then presents the central questions and themes of the thesis, lists its settings, process and methods, identifies its aims and surveys its structure.

1.1 Overview

When my son first learnt to write numbers he would occupy himself for hours with a game of sequences. He would write down a number, then add 2, and repeat until the page was full. Then he would do the same with 3, or 4. Before long, this was not a challenge, and multiplication replaced addition. In due course, this game brought the inevitable question – how far can it go (if we ignore the size of the page)? This story is a familiar one. Yet, for many students, by the time they are confronted with number sequences in a school, this naïve fascination is lost. Occasionally we observe such magical moments of spontaneous mathematical learning. We see children working out of their intuitions, driven by a passion for knowledge, and either developing these intuitions into structured knowledge or challenging and refining them. Yet how can one design for such moments? Furthermore, how can one reify the knowledge of designing for learning as an object for scientific scrutiny and practical utility?

The ability to learn and the passion to do so may be innate human characteristics. Education, on the other hand, concerns identifying specific structures of knowledge and directing learning towards these by assembling resources and activities under cultural, institutional, social, and psychological constraints. In other words, education is designed learning, and as such, an incredibly complex and inherently multidisciplinary endeavour. Any framework attempting to address this domain needs to identify methodological tools which allow it to confront these complexities. Such tools need to balance the need for a crisp directive for action with a rich representation of context, intentions and possible solutions. A framework should supply a range of modelling tools which act as mediating objects in a continuous design level discussion between all stakeholders in an iterative, user-centric development process.

This study originated in my interest in providing children opportunities for learning mathematics. My background in Computer Science led me to relate to this as a design problem, and try to systematise it as such. This attempt gave rise to an enquiry into the prospects and challenges of a design science of technology enhanced mathematical education (TEME), resulting in a multi-layered analysis, reflecting on design as an object, a method and an outcome of study. I see design as a problem-solving activity, and therefore my reflections on design need to be anchored in a particular problem domain. The arguments I make regarding the primary domain of this thesis are evaluated by applying them to a demonstrator domain. The demonstrator domain I chose concerns the use of technology to support learning about number sequences through construction, collaboration and communication.

Three interleaved themes connect the primary and the demonstrator domains: narrative, systematisation and representation. Jerome Bruner argues that narrative is a powerful cognitive and epistemological construct (Chapter Two). I see narrative as a key element in the process of *Situated abstraction* (Chapter Two), forming a path from experience to knowledge. Yet structured knowledge is often perceived as propositional: a formalism which defines terms, states axioms and rules, then derives theorems and proves them. Its structures are static and timeless, devoid of time and person. This would appear to be antithetical to narrative form, which is always personal, contextual and time-bound. To reconcile these forces, we need representations which capture the essence of narrative and transpose it to structured forms.

In the primary domain, this proposition motivates a search for suitable representations for capturing tacit design knowledge subjecting it to scientific process. In the demonstrator domain, it motivates the design of representations and activities by which learners derive mathematical forms of thinking, acting and conversing from their experiences.

The reminder of this chapter expands on these themes. Sections 1.2, 1.3 and 1.4 introduce the design approach to educational research, and develop these themes in its context. Section 1.5 outlines the topic of the demonstrator study, while section 1.6 describes its settings. Section 1.7 notes the research process and methods of the study, section 1.8 lists its aims, and section 1.9 reviews the structure of the rest of the thesis.

1.2 Education as a design science

Putting design in the centre naturally draws me towards the emerging paradigm of design based research in education (Brown, 1992; Noss & Hoyles, 1996; diSessa & Cobb, 2004). Chapter Two traces this tradition back to the seminal work of Herbert Simon (1969). Simon defines design in broad terms: "*everyone designs who devises courses of action aimed at changing existing situations into desired ones*" (Simon, 1969, p. 129), and calls for a discipline of design science. Simon argues that the world we experience is mostly man-made (sic), but that it nevertheless needs to be investigated scientifically. However, such an investigation is radically different than that of the natural world, not least because we cannot detach ourselves from the object of our study. I argue that a design science approach implies a value-laden scientific agenda, a change of methods and awareness of subjective issues such as representation. I present this argument in section 1.4 and elaborate it in Chapter Two.

1.3 Systemisation of design

A quest for the systematisation of design serves two purposes: exposing it to scientific enquiry and opening it up as a public resource. The current discourse of educational design tends to oscillate between two extremes: high abstractions on the one hand and anecdotes on the other. From a scientific point of view, abstractions tend to offer "truisms" which are hard to refute and anecdotes are so specific that they are hard to critique on a theoretical level. From a practitioners' perspective, the abstractions give little practical guidance and anecdotes offer little confidence when their conclusions are transferred to a new problem. A design science of education should be based on a linguistic framework which offers an intermediate level of systematisation, rising above anecdotes but remaining grounded in reality. Such a framework would allow us to capture the structure of educational situations, the challenges they engender, as well as the means of addressing them, in forms which should empower learners and teachers to control their practice as much as it allows researchers to inspect it scientifically.

The empowering nature of systematisation, and its relation to issues of language and communication, links the primary domain of this work with the demonstrator domain. This analogy is demonstrated in the words of Hans Freudenthal, who sees the capacity to systematise as a primary educational goal:

> Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed, with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics. (Freudenthal, 1968, p. 7)

I find this passage inspiring when seeking a systematisation of design. I am perusing an understanding the *activity* of systematising design, not just in its outputs. My own path led from intuitive solutions to problems in designing for mathematics learning to a first sketch of a language of design patterns for TEME. But as much as this sample of design patterns should be useful for others pursuing proximal objectives, my success should be measured in the signposts I provide for others to follow a similar trail, expanding the language of patterns or creating their own.

The notion of narrative is another link between the primary and the demonstrator domains of my work. I see narrative as essential for describing design experiences and processes, as an initial step towards their systematisation. I also see narrative as playing a role in communicating design knowledge to broad audiences. With respect to the problem domain, I wish to design activities which will guide participants along trajectories from their narrative intuitions to a structured understanding of number sequences. An issue I wish to explore is: how can the epistemic power of narrative be harnessed in the construction of systematic knowledge? What is required from such narrative, and what is required from the activity encompassing it? Specifically –

- How can design knowledge be expressed in narrative form, and what is the value of such a representation?
- How can systematised structural representations of design knowledge be derived from narrative forms?

1.4 Design as a method, an object and an outcome of study

The opening section of this chapter highlighted Herbert Simon's (1969) call for a science of design, as a seminal voice in the background of the emerging Design Research paradigm (Chapter Two). This section broadens the perspective on design science and design-based research, raising some methodological questions which lead into the outline of my methodological approach.

Simon distinguishes between the natural sciences and the sciences of the artificial, challenging the view of the latter as 'practical' science or 'vocational arts'. At the core of the study of the artificial, Simon places the science of design (section 1.2). Simon's concept of design science entails more than a shift in the subject of study. It calls for a change in scientific agenda. Whereas natural science is concerned with what *is*, design science asks what *ought to be*. Neurobiology and psychology investigate learning from the perspective of a natural science, while the science of education takes a design stance. The former are concerned with how humans learn, whereas the latter asks how they *ought to learn* and how they *can be helped to learn*. The first may claim to be value neutral and objective, but the questions of education, by their imperative nature, are evidently derived from the observers' (often implicit) ethical, social and community agenda. Sections **Error! Reference source not found.** and 1.3 hint at the values underpinning my work – the aesthetic pleasure and the empowering nature of systematisation. Such values precede the work itself, and thus should be read as underlying axioms rather than arguments to be discussed.

The second implication that Simon (1969) draws from the imperative nature of design science regards the method of problem decomposition. The process of decomposing complex problems into simpler ones is one of science's powerful ideas. Design science is interested in purpose, intent and the shaping of the world to these ends. Therefore, Simon proposes function as the appropriate axis of decomposition. The functional focus also leads Simon to what he calls the generator-test cycle (1969, p. 149) as a viable method of achieving decomposition while acknowledging the networks of interdependencies between components. The design process iteratively generates solutions and then tests them against an array of functional requirements. This cycle maps directly to the iterative process of design-based research.

A focus on function introduces the human agent who interacts with the objects of study. The agent's perception of the object is no less crucial to its function than the nature of the object itself. Ontologically XIX and 19 may be the same, but epistemologically they are radically different: try to compute XIX[ii]. A third key concern of Simon's is the place of representation in science. Whereas natural science strives for representational invariants, design science is deeply concerned with the way problems under investigation are represented in order to illuminate our capacity to solve problems.

The last couple of decades have witnessed the growing popularity of design research as a valuable methodology for educational research. Design based research is a methodology for the study of function. Often referred to as design research or design experiments, it is concerned with the design of learning processes, taking account of the involved complexities, multiple levels and contexts of educational settings. The primary aim is to develop *domain-specific theories* in order to understand the learning process. DiSessa & Cobb (2004) claim that design studies can – and should – make significant theoretical contributions by addressing the gap between theory and practice. They suggest that design research may offer ontological innovations – new constructs for describing and discussing educational phenomena.

The goals of TEME are ambitious, and its challenges complex. Design research offers a sophisticated response to these challenges, but sophistication is often hard to communicate. This difficulty is amplified by the shortage in clear consensual frameworks, as argued in Chapter Two. At its core, a design science of TEME requires an epistemic infrastructure: a set of conventions defining the rules and boundaries of discourse of the scientific community, and the logical system by which claims are presented and justified. This study attempts to make a modest contribution towards such an infrastructure, by proposing several epistemic elements and deriving methodological tools from them.

1.5 The Demonstrator Domain: Learning About Number Sequences

As illustrated in the story at the top of this chapter, children often engage themselves with spontaneous games of number sequences. Many teachers, as well as several national curricula, use games of pattern-spotting in number sequences as an entry into algebra. The challenge arises when one tries to move on from informal games to the understanding of mathematical structure. A number of researchers, including Noss et al (1997) and Radford (2000), point to the difficulties students encounter in shifting from pattern spotting to structural understanding. Students often tend to base their conclusions on superficial or incidental patterns they observe in the sequence, rather than on arguments referring to its structure. I ask what are the known difficulties encountered by students in this domain and how they can be explained. I am interested in the possible contribution of the exploration into number sequences to the construction of advanced mathematical concepts such as function and convergence.

My choice of the demonstrator problem domain, and the specific educational goals of my design, is derived from several parallels with the primary domain of this thesis. The first among these are the values guiding my work. Several writers (Ernest, 2007; 2002; 2000; Sriraman and Steinthorsdottir, 2009) have proposed social justice and personal empowerment as a motivation for teaching mathematics. Papert (1971) challenges the positioning of mathematics as the privilege of a small elite, arguing that education should enable all learners to *be* mathematicians and adopt *mathematical ways of thinking*. Resnick (2007) calls for a focus on creative thinking, as a key for success in today's society. In line with these thoughts, I perceive mathematical creativity as a source of personal empowerment, and see skills of mathematical discourse as vital for a healthy modern society. Yet I hold that the only viable basis for engaging any individual with them is an appreciation of the intrinsic beauty of the system and the ideas of mathematics.

A second parallel between the two domains is the focus on representation. Many educational studies highlight the importance of representation, even if rarely in a direct reference to Simon (with the notable exception of Kafai, 1995). In mathematics, Noss & Hoyles (1996) observe that the issue of selecting and constructing representations is key to learning, and the potentials of alternative representations have been a prevailing concern of the constructionist tradition. These themes are present even in the story that opens this chapter. The child in question constructed his understanding of sequences through an activity, specifically – a game. He would not have been able to do so in the absence of a representational system (Hindu-Arabic numerals) and the technology to support it (pen and paper).

Chapter Six reviews some of the difficulties associated with number sequences, as reflected in the literature. I see many of these difficulties as rooted in a failure to provide learners with representations that would allow them to use their intuitions as a basis for formal concepts, rather than requiring them to abandon them. I claim that the naïve view of number sequences is recursive, expressed as a relationship between consecutive terms rather than its general rule (e.g. describing the sequence represented by the function $f: x \rightarrow 2x + 1$ as "add 2"). If this claim is true it should be possible to design activities which enable children to formalize their recursive intuitions derive a mathematical symbolism from them. Such symbolism, whether its representational form be algebraic or not, should allow children to express and respond to structural arguments about sequences. A central aim of the demonstrator study is to design a set of activities which build upon children's recursive intuitions of number sequences, and encourage them to develop these intuitions into structured formal arguments.

The use of technology in learning environments has become ubiquitous in recent years, taking diverse forms, derived from underlying, often implicit, models of learning. The examples I choose focus on functionalities which are more directly related to the processes of learning, namely construction, communication, collaboration and, eventually, reflection. The design challenge posed by these examples is to produce tools and activities which support social and cognitive processes of learning; supporting the individual construction in the sense of providing stimulating and effective learning activities, and supporting the social process through classroom practices and web-based collaboration systems. Inevitably, the epistemic processes themselves need to be understood to a degree which will enable valid and effective design.

1.6 The research setting

The empirical work of the demonstrator study was supported by the *WebLabs* project, directed by Professors Richard Noss and Celia Hoyles, as detailed in Chapter Five. Some of the analytical work was supported by the of the *Learning Patterns* project[1], directed by Dr. Niall Winters and Professor David Pratt. The WebLabs project explored the collaborative construction and communication of mathematical and scientific knowledge in communities of young learners (age 10-14), by designing new learning activities and the tools to support them. To this end, it employed and enhanced two technologies: the ToonTalk programming environment as a constructionist medium and a bespoke collaborative knowledge-building medium called WebReports. The technological platform used by the WebLabs projects manifests a particular educational approach. The nature of the project was such that the underlying pedagogy and the supporting technology shaped and reshaped each other. Within this context, my individual responsibilities included leading the strand of activities on number sequences and the development of the WebReports system.

The main bulk of the analysis and articulation of generalised design knowledge derived from the empirical work was done after the completion of the WebLabs and Learning Patterns projects. This includes the construction of the main epistemological and methodological arguments. By and large, these were developed by desk research, reflecting on my own field work of that of others.

1.7 Process and methods

The principal concern of this study is a scientific enquiry into the process of techno-pedagogic design in TEME. The main source of data for this enquiry is my personal experiences in such a process, in the problem domain of number sequences. The craft of designing tools and activities for learning calls for domain-specific methodologies, which provide validation of designed artefacts with respect of their intended purpose.

Three constructs play a prominent role in this thesis, with respect to the scientific process of studying design in TEME: design narratives, design patterns, and the cycles of design research in which they are embedded. The first two are representations of domain design knowledge, the latter is a description of the process by which they are derived. The primary study reviews the rationale for

[1] http://lp.noe-kaleidoscope.org, Kaleidoscope JEIRP

these forms and their historical development. It then proposes an operational framework for their use. The demonstrator study applies these constructs within framework in a TEME problem domain.

1.7.1 Cycles of Design Research in TEME

Chapter Four identifies methodological characteristics common to many design studies. Among these are a commitment to practical as well as theoretical contributions, a highly interventionist and agile attitude, and a cycle of iterative research. This cycle includes phases of theory, design, implementation, execution (experiment / practice), articulation of experience, interpretation, evaluation and analysis, and feedback to both theory and design. This cycle is embedded in a meta-cycle, which includes a framing phase, an empirical phase and a retrospective analysis phase.

The demonstrator study was conducted through three cycles of design experiments. Tools and activities were evaluated during trials with small groups of six to ten children in London schools[2]. Analysis of empirical observations provided insights into subjects' learning trajectories and informed the subsequent cycle of design. The three design iterations were followed by a phase of retrospective analysis.

1.7.2 Design Narratives

Design narratives systematise an innate form of extracting knowledge from experience. They provide a first tier of interpretation by affording rich contextualised descriptions of design experiments. Chapter Two makes a case for their appropriateness as an epistemic form in design research, and Chapter Four and Five realises this form as a methodological instrument.

Chapter Six presents the empirical part of my work, as a set of nine design narratives. These narratives are organised thematically according to the three main activities designed and tested in the demonstrator problem domain: basic number sequences, the guess my robot game, and convergence and divergence. Each activity was motivated by specific learning objectives. The design of the activity co-evolved with that of the technological tools that support it, both in terms of the construction environment and in terms of the collaboration platform. The narratives encompass all these elements and trace them chronologically within and across iterations.

1.7.3 Design patterns

Design patterns capture essential features across narratives, encapsulating recurring challenges and forces pertaining to a domain of learning design, the interactions between them and possible methods of solution. Design patterns are seen as *situated abstractions of design knowledge*: they attempt to capture generic principles, while acknowledging the limits of genericity and our epistemic need to maintain a link between systematised concepts and the context from which they were derived. A design pattern encapsulates a problem, the context in which it arises, and a possible solution based on examples of practice. This structure promotes a functional decomposition of the problem domain, and offers a representation of design knowledge which yields itself to theoretical scrutiny as well as pragmatic implementation. Thus it appears to be a good candidate form for a design science of education. However, to qualify as science, the process of deriving patterns from

[2] Classroom trials were assisted by my colleague Gordon Simpson.

case narratives needs to be transparent and the patterns themselves need to be rigorously validated. Chapter Two reviews the origin of and motivation for design patterns, and traces their trajectory from architecture through software engineering to educational research. Chapters Four and Five propose criteria for adapting design narrative as a scientific form, and a structured process for extracting patterns from narratives.

Chapter Eight presents a set of seven detailed design patterns, and outlines seven more. Together with the design narratives in Chapter Six, these form an initial draft of a pattern language for TEME.

1.8 Aims of this study

Having presented the context of my research and the key issues it explores, I now have the means to define the scope and objectives of my work. The overarching intent of this work is to contribute to the understanding of learning as a design science, highlight the implications of such a paradigm, and propose ways to theorize design in a manner which draws on and informs educational research.

This intent is manifested on three levels: epistemic, methodological and pedagogical. I aim to:

- Identify potential elements of an **epistemic infrastructure** for a design science of TEME.
- Combine and elaborate the elements identified into a coherent **methodological framework** in a given research TEME context.
- Apply the methodology in the problem domain of learning about number sequences and demonstrate its potential by producing a contribution towards a language of **pedagogical patterns** for TEME.

These aims are elaborated in Chapter Three, drawing on the foundations laid in Chapter Two.

1.9 Structure of Thesis

The present chapter illuminated the main themes and approaches which guide this study and enumerated its aims. These aims build on each other, and together form a multi-level argument which runs through the thesis. Figure 1 provides a high-level map of this argument. The rest of the thesis works systematically through it.

My point of departure is the intuitive observation differentiating learning, as a spontaneous indigenous human capacity, and education, as a complex and challenging realm of design. This observation provides motivates the question that inspires my work: how to design for “magical moments of learning”? Reformulated as a subject for scientific enquiry, this question is translated into a cascaded study of design in TEME: first, as an epistemological question, leading to a methodological question, and evaluated by applying the outcomes of the first two investigations to a pedagogical challenge.

The initial review of the field of design-based research in TEME identifies a lack of consensual epistemic infrastructure. This motivates the first aim of my thesis: to identify elements of such an infrastructure. Three constructs emerge from a review of existing approaches, in the field of education and beyond: the cycles of design research, and the representational forms of design narratives and design patterns. Recognising the need to operationalise these abstract constructs leads to the second aim of my thesis: identifying a possible methodological framework which utilises these constructs. This aim is addressed by situating the epistemic elements in a concrete research setting. First, the setting is described. Then appropriate methods enumerated for collecting data, organising it into design narratives, deriving design patterns from these and inducing theoretical contributions from the resulting sketch of a pattern language.

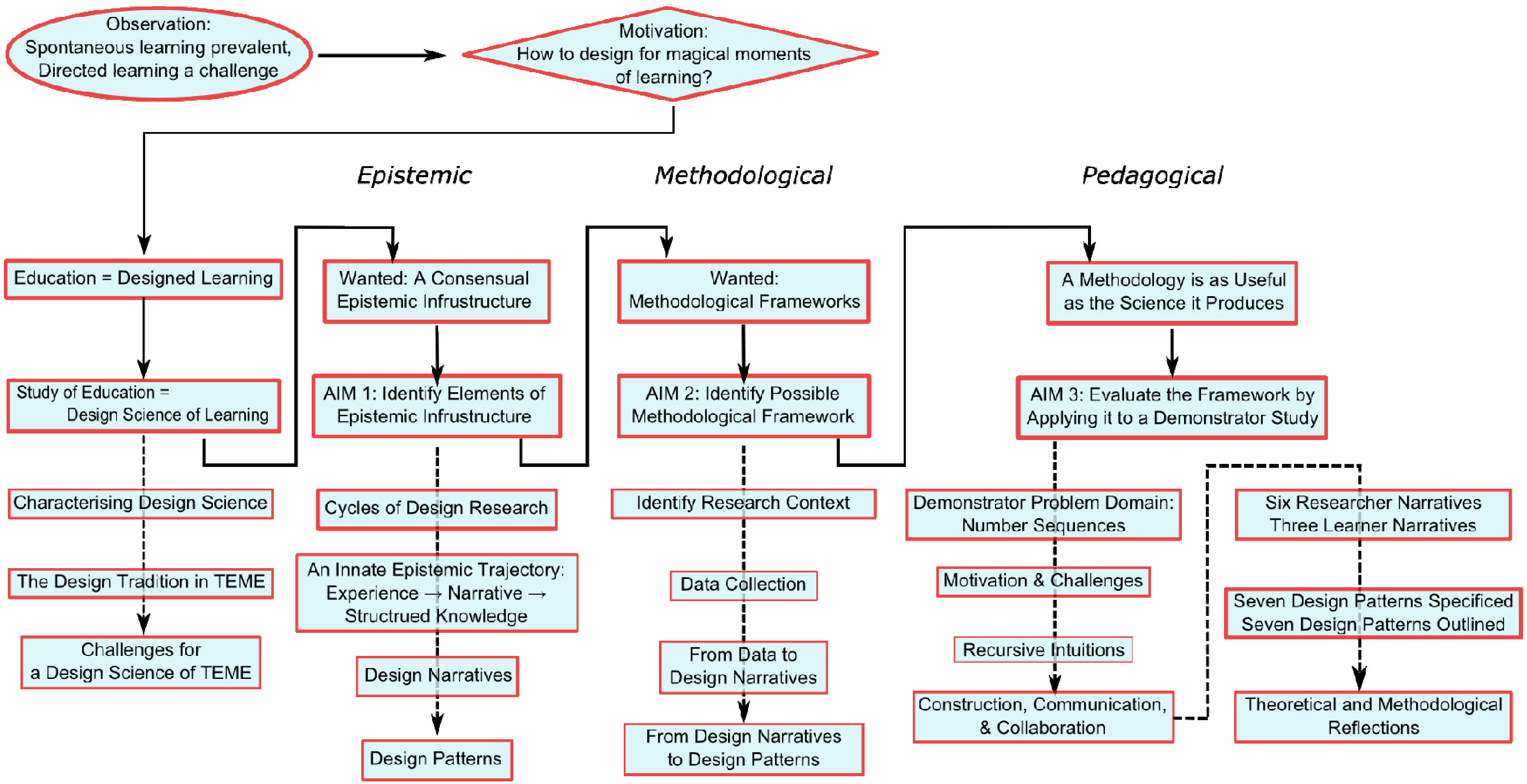

Figure 1: a birds-view map the argument of this thesis. Inspired by a desire to design for learning, the theory first contemplates the prospects of education as a design science of learning, and then traces the implications of this proposal through three levels: epistemic, methodological and pedagogical.

Chapter Two provides the theoretical backdrop, by reviewing the traditions of design research in education and in related fields.

Chapter Three builds on the foundations provided by Chapter Two to list the aims of the thesis in detail.

Chapter Four presents the three central constructs and delineates the conditions for their use as scientific instruments.

Chapter Five describes the methodology for the demonstrator study, by projecting these three constructs into the specific research settings.

Chapter Six reviews the challenges and prior research in the demonstrator domain, deriving several conjectures which will underlie the design of activities and technologies.

Chapter Seven presents a set of nine design narratives, recounting the design experiments conducted in the demonstrator domain.

Chapter Eight derives a set of design patterns from these narratives, seven of which are articulated in detail and seven outlined.

Chapter Nine concludes the thesis by discussing the major findings and mapping them to the aims.