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Version 0.92, 13 Nov. 09

Chapter 3 Aims and Method of Enquiry

This chapter revisits the aims outlined in Chapter one, in light of the review presented in Chapter 2. These aims are elaborated to propose a programme of research, and consequently characterise the overarching method of enquiry.

3.1 Introduction

Chapter Two presented an argument for a design science of technology-enhanced mathematics education (TEME), and reviewed existing traditions in this field. A key issue which emerged from this discussion was the need for a clearly articulated consensual epistemic infrastructure for this paradigm. This issue gave rise to several questions. The thesis intends to make a contribution towards such an infrastructure, by acknowledging existing frameworks, identifying gaps within them, and proposing some constructs to address them. The resulting framework is tested by applying it to a genuine problem domain, namely the use of technology to foster learning about number sequences through construction and collaboration.

Chapter One opened with the characterisation of education as designed learning, thus establishing a multi-faceted link between design and epistemology, or the creation of knowledge. This link is a motif that flows through this thesis. Design emerged as a method of study, an object of study and an outcome of study. The link to epistemology replicates these layers. Epistemology can be read through three lenses: genetic, normative and generative; how we construct knowledge, how knowledge should be produced, and how we can design for the creation of knowledge. Piaget notes that “For many philosophers and epistemologists, epistemology is the study of knowledge as it exists at the present moment; it is the analysis of knowledge for its own sake and within its own framework without regard for its development” (Piaget, 1970, online). By contrast, “Genetic epistemology attempts to explain knowledge, and in particular scientific knowledge, on the basis of its history, its sociogenesis, and especially the psychological origins of the notions and operations upon which it is based” (ibid). The shift from the universal to the anthropocentric is completed by the school of design research in education. In his monograph *Toward an Epistemology of Physics,* diSessa posits: "A theory of knowledge and its development ought to be significant for education." (diSessa, 1993, p205). Epistemology has been repurposed from the study of knowledge as an abstract universal, to its evolution in the individual, social and historical human context, and finally to the question of how such dynamics can be facilitated and perfected.

The pragmatist nature of a design stance to educational research suggests a tight dependency between the three levels of epistemology: the method by which we study education (its normative epistemology) needs to link our understanding of how people learn (genetic epistemology) to how we design artefacts for learning (generative epistemology). The specific aims of this thesis are derived from this realisation. The aims identified in Chapter one can now be situated in this context and elaborated in view of the observations from Chapter Two. An overarching theme of this thesis is to:

Consider the study of education as a design science. Highlight the implications of such a paradigm, and propose ways to theorize design in a manner which draws both on educational research and computer science.

Chapter Two has taken the first steps in this direction, by proposing a characterisation of a design science of education, and arguing the case for its necessity. At the same time, that chapter has identified several issues which need to be addressed for the promise of a design science of education to be realised. These issues relate to the (normative) epistemic infrastructure of the paradigm, and its ability to link genetic to generative epistemologies. Consequently, the aims of this thesis address the question of a design science of TEME at three levels: epistemic, methodological and pedagogic.

Aim 1: *To identify potential elements of an* **epistemic infrastructure** *for a design science of technology enhanced mathematics education*.

Chapter Two identified the need for articulating an epistemic infrastructure for a design science of TEME. Such an infrastructure should delineate the paradigm's commissive space (Kelly et al, 2008) and argumentative grammar (Cobb and Gravemeijer, 2004). The former refers to the explicit and implicit rules and assumptions which bound the discourse of a scientific community, and the latter to the logical system by which claims are presented and justified, independently of their content. Chapter Two also identified some desired qualities of this epistemic infrastructure. It should be -

- **Accessibile** to the scientific community, practitioners and policy makers.
- **Transparent and traceable** so that the full cycle of a design study should be observable by an external reviewer.
- Sufficiently **expressive** to allow articulation of all that is needed to support the above requirements.
- Organised with a **functional-pragmatist orientation,** indexing findings by the problems they solve rather than the resources (physical or theoretical) they utilise.
- Promote **Cumulativity** and afford easy aggregation of knowledge, building new results on the basis of prior art.

This thesis aims to explore two forms of reporting and analysing design experiences in education which claim to afford systematisation and effective communication of design knowledge in this domain: design narratives and design patterns. The theoretical issues arising from Chapter Two have practical implications in terms of the craft of research. The cycles of design research, and the embedding of Design narratives and design patterns within them, are at the apex of the forms of practice and associated representation discussed in Chapter four.

Aim 2: *To combine and elaborate the elements identified into a coherent* **methodological framework** *in a given research context*

In order to operationalise the proposed elements of epistemic infrastructure, the forms of design narratives and design patterns need to be embedded in the cycles of design research and appropriate tools of data collection and analysis recruited to support the mechanics of conducting experiments, collecting design narratives, eliciting patterns, and developing and substantiating theoretical innovations.

In specifying these tools and procedures, the epistemic infrastructure is projected into a methodological framework. The details of such a framework are contingent on the specifics of the

research setting. This thesis takes the learning of number sequences in lower secondary level as its application domain. The experimental setting involved small groups of children in informal activities adjacent to their school, using the ToonTalk programming environment as a constructive medium and the WebReports collaboration system as a communicational medium. This thesis aims to present, justify and evaluate a methodological framework based on design narratives and design patterns which is appropriate for this setting, and consider the breadth of its applicability to similar situations.

Aim 3: *Apply the methodology in a problem domain and demonstrate its potential by producing a contribution towards a language of* **pedagogical patterns** *for technology enhanced mathematics education*

Aim 1 and Aim 2 relate to the nature and the process of a design science of education. The claims arising from these discussions need to be judged both logically and empirically. The empirical test of a scientific programme is in its execution. Thus, Chapters Six, Seven, and Eight constitute a "mini-thesis", employing the outcomes of the previous chapters as a framework for a design study in a demonstrator problem domain. The fruits of this enquiry should, in a modest way, shed light both on processes of learning (genetic epistemology) and on effective ways of facilitating such processes (generative epistemology). Reflections on the process and outcomes of this enquiry should provide initial evidence towards evaluating the normative claims in Chapters Two and Four. The outputs of the design study will be captured as set of design narratives and design patterns, contributing towards a (pedagogical) pattern language for TEME. This contribution will be assessed in terms of its theoretical and practical potential.

Sections 3.2, 3.3 and 3.4 enumerate some questions elaborating these three aims, and section 3.5 outlines the method of enquiry by which they will be pursued.

3.2 Notes on Aim 1: Towards a Design Science of Technology Enhanced Mathematics Education

Chapter Two noted the distinction between asking "*how do people learn*" and "*how do we provide the conditions for people to learn*". While the former could be investigated from a perspective of natural or social sciences, the latter suggests a design science stance. Several immediate questions emerge from this observation:

- What is the nature and character of a design science of mathematics education?
- What relationship between educational research and practice is implied by a design science perspective, and what advantages does it bring to both?
- What are the epistemological structures of a design science of mathematics education – how is knowledge sought and truth established?

These questions were considered in Chapter Two. The emerging conclusions provide the foundations for the rest of this thesis, and at the same time act as conjectures to be explored empirically. If the arguments in Chapter Two are valid, then it should be possible to conduct a study along the lines sketched there which would produce innovative results and offer a significant contribution in terms of both practice and theory. Chapter Four addresses this objective.

The review of the possibility of education as a design science, its motivation and current state gave rise to several observations regarding the epistemic infrastructure of this paradigm. The main question arising is: does there exist an epistemic infrastructure that satisfies the requirements listed, and if it does, will it answer the critique of current practices in design research of mathematics education? Specifically –

- What are the existing conventions regarding appropriate forms for conducting design-based research in education?
- What are the requirements on methodological tools implied by the characterisation of the design paradigm? How do current tools map to these requirements? Can they be combined and enhanced to provide a more comprehensive coverage?
- Which new tools can be incorporated to address the issues noted and how?

Chapter Four considers these questions and presents some contributions towards an epistemic infrastructure. These are used by Chapter Five to propose a concrete methodological framework. These observations are related to issues pertaining to the impact and prospects of the underlying paradigm. Indeed, the demonstrator study reported in Chapters Six, Seven and Eight provides a test of a design science of education as reflected in the interpretation embodied in this framework. The primary innovation in this framework is the combination of design narratives and design patterns as a means for analysing and reporting design knowledge in mathematics education. The specific contribution in terms of these structures is the focus of Aim 2.

3.3 Notes on Aim 2: Towards a Pattern-based Methodology for a Design Science of Technology Enhanced Mathematical Education

A consideration of the epistemic infrastructure for design research of TEME, in response to Aim 1, suggests the combination of design narratives and design pattern as a contribution towards an epistemic infrastructure for a design science of mathematics education. This proposal rests on several claims regarding the qualities of such representations, when embedded in appropriate research practices, namely that they -

- Allow the process and outcomes of techno-pedagogical design to be captured in a manner which affords analysis, critique, and refinement.
- Provide an intermediate level of abstraction which links theory to practice and facilitates a design-centric discussion of the problem domain.
- Combine narrative and structured representation, linking intuitive and rich descriptions with rigorous semantics.

Chapters Two, Four and Five provide theoretical warrants for these claims. Empirical evidence is required to further substantiate them. Such preliminary evidence, as a proof of concept and feasibility, is presented in Chapters Seven and Eight in the form of a contribution towards a pattern language in the domain of TEME. This language is to include a system of design patterns and supporting design narratives, with theoretical and practical implications. It intends to –

- Capture and link theoretical and practical innovations in a manner that is transferable to other circumstances, while clearly delineating the boundaries of transferability.

- Articulate these innovations in a form that is accessible to both academic and practitioner communities.
- Uphold a scientific standard of transparency and traceability.
- Afford cumulativity, by providing forms for integrating other relevant bodies of knowledge and encapsulating outputs in a form that can be integrated and reused elsewhere.

3.4 Notes on Aim 3: Design Patterns for Learning about Number Sequences by Construction, Communication and Collaboration

In order to gauge the value of the epistemic constructs and the methodological framework emerging as a response to Aims 1 and 2, they need to be applied to a genuine problem domain. The problem domain chosen for this thesis is the design of tools and activities for learning about number sequences, in an extra-curricular lower-secondary school setting. Although the main effort in this thesis is oriented towards general theoretical, disciplinary and methodological questions, the demonstrator study should yield meaningful, if modest, results in its domain.

The demonstrator study reported in Chapters Six, Seven and Eight follows groups of young teens exploring questions about number sequences through collaborative construction activities. These activities utilise two integrated computational media: WebLabs, a web-based collaboration system and ToonTalk, a game-like programming environment. Consequently, the theoretical and practical contributions derived from these experiments, and expressed through design narratives and design patterns, should bear immediate relevance to the problem domain. Some of the outputs will be specific to the domain of learning about number sequences through construction and collaboration, and some will have a wider remit.

The demonstrator study will:

- Identify obstacles to learning in the domain.
- Raise pedagogical conjectures addressing these obstacles.
- Design tools and activities manifesting these conjectures.
- Evaluate the success of these tools and activities.
- Capture the insights emerging from the evaluation in the form of design patterns.
- Note how the outcomes reflect back on the initial pedagogical conjectures.

3.5 Method of Enquiry

The three aims declared in Chapter One, further detailed in section 3.1, and elaborated in subsequent sections, approach the question of research in mathematical education on three levels of abstraction. The first considers the epistemic level, the next explores methodological issues, and the third level engages in concrete pedagogical issues. These three levels are linked in two directions: the arguments in each level form the basis for the next, and the evidence observed at each level feeds back to support the previous one. Thus, each aim applies a different method of enquiry, while relating to the others.

Aim 1 is investigated through theoretical review and discussion, with an emphasis on constructing a coherent argument by critical consideration of the existing literature.

Aim 3 is addressed through a cycle of experimental field work, analysis and reflection.

Aim 2 mediates between these two by deriving and projecting the consequences of Aim 1 into the specific settings of Aim 3, monitoring the effectiveness of the derived constructs and adjusting them in response to the empirical observations.

The multi-layered agenda of this study leads to a recursive structure, of a "thesis within a thesis": an outer sphere constructing the theoretical and methodological frameworks, and an inner sphere of an empirical study within these frameworks.

3.6 Conclusions

This chapter defined the aims of the thesis and elaborated them in view of the theoretical review presented in Chapter Two. Each aim was expanded into a set of specific research questions. Each aim addresses the notion of research in technology enhanced mathematics education on a different level, while referring to and feeding back into the other levels. Consequently the methods of enquiry differ between levels while being tightly interlinked.